A Behavioral Theory of Discrimination in Policing*

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Abstract

Racial disparities in policing are well documented. In addition to officer animus towards some groups ("taste-based discrimination"), these could be driven by officers' beliefs that crime rates are higher in some communities ("statistical discrimination"). But where do these beliefs come from, and what if they are incorrect? We analyze a formal model where officers form beliefs using crime statistics, but make a common inferential mistake by not fully adjusting for the fact that they will detect more crime in more heavily policed communities. This creates a feedback loop where officers (incorrectly) believe there is relatively more crime in communities that are policed intensely, leading to persistent over-policing. We also show that discrimination driven by false beliefs is contagious across officers. This means that inferential mistakes can exacerbate discrimination even among officers with no animus and who sincerely believe disparities are driven by real differences in crime rates.

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Police departments in the U.S. have become more professionalized over the past half-century, aiming to reduce arbitrary and abusive policing practices. And yet, dramatic racial disparities in policing persist. For example, 80% of people stopped under New York City's now-defunct "stop and frisk" policy were either black or Latino despite the fact that those two groups make up only half of the city's population (Goel, Rao, and Shroff 2016). In Boston, blacks comprised 63% of police stops that did not end in arrest from 2007 to 2010, even though only 24% of the population is black (The Sentencing Project 2015). Non-white motorists are more likely to be stopped than white motorists (Epp, Maynard-Moody, and Haider-Markel 2014).

There are two standard theoretical explanations for these disparities, one driven by preferences and one driven by beliefs. In a purely preference-driven account—often called taste-based discrimination—officers intrinsically like being punitive towards some groups, or dislike being punitive towards others. The second explanation—typically called statistical discrimination—is that there are real differences in the rates of criminal behavior across groups. Knowing this, police allocate more time towards groups who commit more crimes, or at least in areas where those groups are concentrated.

Another explanation for policing disparities sits somewhat uncomfortably between these two standard explanations. What if officers police certain groups more intensely because they believe that the group has a relatively high crime rate, but this belief is incorrect, or at least exaggerated (Glaser 2015)? In a proximate sense, this is discrimination driven by beliefs. But to the extent that these beliefs are incorrect we might suspect this inaccuracy to be driven by intrinsic dislike of the group. If so, it may not make sense to think of the belief and preference channels as distinct and separable causes of discrimination since they may affect each other.

This is not just a hypothetical. There are many reasons to doubt that the relevant decision-makers always have correct beliefs about different groups. Even though police departments have become more data-driven in recent decades, police officials typically need to make decisions under time pressure without the benefit of the kind of statistical expertise that would enable high quality assessments about crime across communities. Indeed, the shift toward data-driven policing has been controversial. If departments rely on either faulty analysis or bad data, this can perpetuate disparities (see, for example, Harcourt 2007; Lum and Isaac 2016). Even the federal courts have weighed in to criticize flawed data analysis by police (e.g., *Floyd v. New York*, 959 F. Supp. 2d 540, S.D.N.Y. 2013).

We develop a set of models which allows for incorrect beliefs about the prevalence of crime among members of different groups. However, we do not allow for *any* arbitrarily incorrect beliefs, and instead explicitly model how and why beliefs become incorrect via a specific inferential mistake. Officers in our models form beliefs about the relative prevalence of crime among members of different social groups based on the number of crimes the police detect among members of those groups, without fully accounting for the fact that more crimes are detected among members of groups that are policed more intensely. We call this *non-conditioning bias*.

Our models also allow for both the possibility that police officers have a taste for discrimination, and the possibility that crime rates are different across groups. In the special case where officers form correct beliefs, these two mechanisms independently affect policing disparities, as in the standard accounts. However, once officers exhibit any non-conditioning bias, this creates a feedback channel where groups who are policed more intensely are viewed as having higher crime rates than they really do. This feedback loop amplifies whatever policing disparities would exist in the absence of non-conditioning bias. Put another way, a taste for discrimination causes inaccurate statistical discrimination.

We first formalize this argument in a simple and analytically tractable model with just one officer. Next, we extend the analysis to include multiple officers. When each officer has correct beliefs (as in standard models), all of their policing decisions are independent and neither officer's preferences or behavior affect the other. However, if one officer has a non-conditioning bias, then their beliefs will be influenced by the behavior of other officers. As a result, the discriminatory behavior of one officer can spill over and cause the other officer to discriminate too.

A straightforward implication of the theory is that faulty data analysis by police departments may unwittingly exacerbate disparities. For example, if departments use data-driven algorithms to predict where crime is likely to occur (Collins 2018), the predictions generated by these algorithms may be highly discriminatory if they are based on simple counts of prior crimes detected by police. Relatedly, since our results predict larger policing disparities than due to statistical and/or taste-based discrimination, we provide a theoretical basis to presume that administrative data on policing is biased (Knox, Lowe, and Mummolo 2019).

Our model suggests that policy responses to discriminatory policing should focus—at least in part—on alleviating distortions caused by non-conditioning bias. Most obviously, equipping decision makers in police departments with appropriate statistical training could help them avoid making faulty inferences from crime statistics. That said, success of a policy like this requires specific individuals to consistently and correctly apply this training even in the face of other professional (or even political) pressures. A more institutionally oriented policy response would focus on ensuring that policing decisions are not endogenous to the data generated by those decisions. For example, departments could establish fully independent crime analysis divisions that are barred from using data generated from policing, such as arrests.[1] Finally, if accurate analysis of crime data remains difficult or infeasible, simply reducing policing intensity in highly policed communities—even if it seems less effective—can help department decision makers to form more accurate assessments of relative crime across all communities.

1 Explaining Policing Disparities

Policing disparities have been well documented.[2] And, there is convincing evidence that statistical estimates of policing disparities may actually understate the extent of those disparities (Knox,

1. Note that our model suggests this separation could be useful even absent other concerns about departments' incentives to misreport data.

2. For a list of studies documenting racial disparities in the criminal justice system (including in policing), see Balko (2018).

Lowe, and Mummolo 2019; Knox and Mummolo 2020). Policing disparities also have important social consequences. For example, they may reduce political participation (Weaver and Lerman 2010)[3] and increase overpoliced populations' contact with the criminal justice system (Glaser 2015).

While the empirical question about whether policing disparities exist is mostly settled, the theoretical question about why these disparities exist is not. There is a robust literature devoted to cataloguing and teasing out the explanations for racial disparities in policing, as well as in the labor market and politics (for example, see Knowles, Persico, and Todd 2001; Anwar and Fang 2006; Glaser 2015; Persico 2009; Doleac and Stein 2013; Ewens, Tomlin, and Wang 2014; Butler and Broockman 2011; Broockman and Soltas 2018; Nathan and White, Forthcoming).

Identifying the underlying causes of disparities is not only an academic exercise; there are important legal and policy implications. Under U.S. law, it is typically impermissible for the government (including police departments) to discriminate on the basis of membership in a protected class, such as race, gender, religion or national origin (see *Floyd v. New York*, 959 F. Supp. 2d 540, S.D.N.Y. 2013). However, in light of geographical patterns in both residential segregation and crime, policing disparities may emerge even when police departments use "facially neutral" (and legal) policing practices, such as deploying resources to high crime locations.[4] For policymakers and police departments seeking to reduce policing disparities, it matters why those disparities exist. Different root causes call for different responses, from firing prejudiced officers and conducting bias training to changing policing tactics and reducing enforcement activities for certain kinds of crimes.

There are two "standard" explanations for disparities, both of which our model captures. The first has its origins in the theory of discrimination articulated by Becker (1957). According to this

3. However, Walker (2020) finds that "proximal contact" with criminal justice system (e.g., via a relative) is associated with increased political participation, and Peyton, Sierra-Arévalo, and Rand (2019) find certain kinds of positive, non-enforcement policing actually increase willingness to cooperate with police.

4. For example, in *Floyd*, Judge Scheindlin notes: "I recognize that the police will deploy their limited resources to high crime areas. This benefits the communities where the need for policing is greatest."

explanation, if police officers have animus toward some groups, this may directly influence where they want to focus their efforts. We capture this in our models by allowing for the possibility that police officers get higher marginal utility for detecting crimes among one group.

The other standard explanation for disparities emerges from the fact that group identity may be informative about crime. This is the mechanism driving the models of statistical discrimination that have emerged from the seminal work by Phelps (1972) and Arrow (1973). For example, if crime rates are different across groups, then given certain kinds of policing objectives (like reducing crime), it would be "rational" to police those groups with different intensities. A simple version of this argument might entail something like this: *police officers pull over non-white drivers at higher rates than white drivers because they believe that non-white drivers are more likely to be carrying contraband* (see Knowles, Persico, and Todd 2001; Anwar and Fang 2006). In this account, police officers may have no animus toward non-whites (or any preference for whites), but are simply trying to maximize the amount of contraband they detect using the information available to them.

Of course, the key question for this explanation is why a decision-maker has those beliefs in the first place. In standard models, an individual's beliefs must be correct. That does not imply that they cannot face any uncertainty. However, they must correctly assess this uncertainty in light of their experiences and environment. For example, an officer can only believe non-white motorists are more likely to carry contraband if the information they possess—processed according to Bayes' rule—indicates that non-white motorists in fact are more likely to carry contraband.

The typical requirement that beliefs be correct does not reflect misguided adherence to a fictional view about how humans think. If we let beliefs be incorrect in arbitrary ways, unsupported by anything the decision-maker actually experiences, nearly any behavior can be trivially explained by assuming actors have a false belief that their behavior is in their best interest. Moreover, the requirement of correct beliefs—while stringent—has generated many new insights about the persistence of inequality between groups. A core contention of much of this research is that statistical reasoning of this kind can generate perverse outcomes with self-reinforcing and discriminatory stereotypes. Coate and Loury (1993) provide an early account of racial discrimination in labor markets, and similar findings explain racial disparities in both robberies and homicide (O'Flaherty and Sethi 2019), as well as the persistence of social segregation (Chaudhuri and Sethi 2008). More generally, the extent to which "top brass" within policing institutions are able to accurately assess the effectiveness of policing practices is a key concern and a potential explanation for policing disparities (see McCall 2019).

While models with arbitrary beliefs typically provide little analytic traction, assuming perfectly correct beliefs limits the range of formal theories of discrimination in a way that does not accord with many scholars' intuitions about how beliefs fuel discrimination. Bohren et al. (2019) survey the broader economics literature on discrimination and find that few papers consider, let alone test for, the possibility that disparities due to statistical reasoning may be based on biased or otherwise incorrect beliefs.

Our broader contribution is to provide a formal theory of policing that allows for some limits on cognition in complex environments but that does not abandon the core contention that beliefs should be generated systematically from experiences. Specifically, we relax the stringent assumption that police officers must have perfectly correct beliefs, and allow for them to have behavioral tendencies. By "behavioral" we mean that they use mental shortcuts when processing uncertainty in a complex environment. Our work is therefore tied closely to a large theoretical literature in behavioral economics focused on the ways in which decision makers deviate from standard assumptions in formal analysis (see Dhami 2016). Our focus is on one kind of behavioral tendency: a decision maker's limited ability to condition on all relevant information—and specifically policing intensity—when making inferences about crime rates. This is non-conditioning bias.

Closest to the non-conditioning bias we consider, Eyster and Rabin (2005) study the phenomenon of the "cursed equilibrium," which can arise in Bayesian games. The model features a player who does not fully account for the fact that other players' actions reflect those players' private information (and are thus informative). In other words, this player does not account for the fact that there will be selection bias in the other players' actions since those other players have incentives to choose different actions depending on their types. In our model, officers make a similar mistake by not accounting for selection bias when making assessments about which communities are high crime communities.[5]

Of course, we are not the first to suggest that cognitive biases are important for understanding policing. In fact, Eckhouse (2019) argues that work on bias in policing overemphasizes cognitive biases relative to structural factors which put police disproportionately in contact with certain communities, with evidence from a change in the stop-and-frisk policy in New York City. Our model highlights that cognitive and structural biases are not necessarily competing explanations, as they can be mutually reinforcing sources of disparities.

Closest to our argument, Glaser (2015) argues that racial disparities in policing are fueled by a feedback loop driven by cognitive biases. Our formalization of this process complements and builds on this work in several ways. First, while Glaser (2015) identifies incorrect beliefs[6] as a cause of racial disparities in policing, we provide a full theoretical account for how these incorrect beliefs can be formed endogenously by policing. In our model, officers do not necessarily start with incorrect beliefs, and must form them solely via their experiences policing. This suggests that rather large policing disparities need not come from outside influences, such as media portrayals of non-whites as criminal. (Although, these may amplify them further.) Second, we show how discrimination can become contagious and spread to other officers who may not have discriminated if they were acting on their own. And finally, we make a broader contribution to the literature on discrimination by highlight that taste-based and statistical discrimination are not distinctly separate explanations for disparities, except under the extreme, knife-edged assumption that officers have

5. Non-conditioning bias also has some similarity to the well-known behavioral phenomenon of base rate neglect (Kahneman and Tversky 1973), though base rate neglect is about underweighting prior information relative to new information, rather than different kinds of information which an officer analyzing crime data acquires at the same time.

6. Glaser (2015) uses slightly different terminology, calling these "stereotypes." We avoid this terminology because much of the formal literature on stereotypes features *correct* beliefs, even though those correct beliefs usually generate bad outcomes (e.g., Coate and Loury 1993; O'Flaherty and Sethi 2019).

correct beliefs.

Finally, a broader intervention we aim to make in the study of discrimination and prejudice (especially outside of formal theory or economics) is to point out that this work frequently blurs the line between preferences and beliefs as drivers of disparities. Canonical work classifies it as prejudice when a person holds an inaccurate belief that members of a certain group disproportionately engage in undesirable behaviors (e.g., commit more crimes) or have undesirable traits (e.g., are more "criminal").[7] In contrast, a key feature of our model is that we clearly distinguish the potential mechanisms driving disparities—preferences and beliefs—and then demonstrate how animus toward a group (preferences) can *endogenously cause* overly negative and incorrect beliefs about that group via non-conditioning bias.

2 Model of a Unitary Officer

We start with a model of a single police officer (pronoun "he"), who we primarily interpret as a high level official who makes decisions for the department as a whole, such as the chief of police. The officer makes a choice about how to allocate resources toward policing two groups, A and B. While we do not introduce notation for the group size, the model is easiest to interpret as one where the two groups are equally numerous.[8]

The officer has a unit of resources, which we primarily interpret as time, to allocate between policing the two groups. Let w_A represent the share of time spent policing group A, with $w_B = 1 - w_A$ left for group B. We assume that the officer can choose to allocate his time evenly between the two groups, but can also choose to police on group more than the other. However, the officer

7. Consider the famous (and oft cited) definition of prejudice provided by Allport (1954): "Ethnic prejudice is an antipathy based on a faulty and inflexible generalization" (p. 9). Key to this definition is the idea that pure antipathy and beliefs about other groups are intertwined. As Katz (1991) points out: "[According to Allport,] What separated a prejudice from other negative social attitudes was, first, the inaccuracy of the belief component, which presumably was a consequence overgeneralization from a set of limited observations." (p. 131).

8. If one group is much larger, then all things equal we would expect the police to spend more time policing that group. The disparities of concern are really with respect to time spent policing per individual. Accounting to this would add complexity to the model without obviously changing our results.

can't choose to allocate *all* of his time to one group or the other. Formally:

Assumption 1. *The officer chooses* $w_A \in [\underline{w}, \overline{w}]$*, where* $0 < \underline{w} \leq 1/2 \leq \overline{w} < 1$.

In addition to being realistic (as we discuss below), both aspects of this assumption—that equal policing is feasible and only policing one group is infeasible—reduce the number of cases to consider for some of our results.[9] We also introduce terminology for the important case where the officer polices one group as much as is feasible:

Definition 1. *If the officer's policing allocation is at either bound* ($w_A \in \{\underline{w}, \overline{w}\}$)*, we say he engages in "extreme policing."*

In the United States, it is typically illegal for governments (including police departments) to target individuals solely on the basis of their social grouping, such as their race, religion, gender, etc. Thus, one way to think about the choice in our model is that the police department decides to target resources toward different geographical locations, which due to residential segregation, have different proportions of the two groups. Unless geographic segregation is absolute, sending officers only to some geographic areas won't completely prevent officers from coming to contact with individuals from both groups. Due to this, the department always has some leeway in determining how much officers come into contact each group, but can't choose to allocate all of their time to one or the other. In Appendix A, we provide a brief discussion of a microfoundation for our model in which the officer chooses how to allocate time between neighborhoods, and not between social groups.

We assume that the allocation of policing effort, as reflected by w_A, affects the detection of crime. As a result, our model is less applicable for crimes that are universally (or near universally) reported, such as murder. Formally, we let the amount of crime caught among members of group J be $c_J = p_J w_J$, where $p_J > 0$. The simplest way to interpret this is that p_J represents the average

9. For example, ruling out $w_A = 0$ or $w_A = 1$ removes corner solutions where the officer spends no time policing one of the groups and, as a result, believes that group commits no crime.

number of crimes committed by members of group J per unit of time, and w_J represents how much time is spent policing this group. This is the data that the officer uses to determine how to allocate his time. In Appendix E, we analyze a variant where the number of crimes caught is not linear in w_J, which complicates the interpretation of the parameters, but does not fundamentally change our argument.

Preferences We assume that objective of the officer is to catch crimes. To capture the notion that the officer might have a taste for discrimination, we allow him to prefer catching crimes among one group or the other. We also assume that there are diminishing returns to the amount of crime caught within each group. This assumption is a reduced-form way to capture the notion that some crimes are "more important" to detect than others, and that the officer will first dedicate time to detecting the more important crimes (within each group).

In Appendix D, we consider more general preferences to capture these ideas, but in the main analysis we use following utility function:

$$u(c_A, c_B) = t_A\sqrt{c_A} + t_B\sqrt{c_B} = t_A\sqrt{p_A w_A} + t_B\sqrt{p_B(1 - w_A)} \tag{1}$$

where $t_A > 0$ and $t_B > 0$ each represent the officer's "taste" for catching crimes among group A and B, respectively. We formally capture the possibility that an officer has animus toward group J by allowing his taste for catch crimes among group J (t_J) to be higher than his taste for catching crimes among the other group.

Definition 2. *The officer has* ***animus towards group*** A *if* $t_A > t_B$, *and* ***animus towards group*** B *if* $t_B > t_A$. *He has no animus if* $t_A = t_B$.

We purposefully use the term "animus" instead of the more common term "prejudice" to refer to the component of our model that generates discrimination via preferences. Use of the term "prejudice" in prior research (and in popular discourse) frequently conflates the preference and belief mechanisms, and we want to keep these separate in our model's primitives. However, our analysis will demonstrate that there is an endogenous link between the two mechanisms. As we discuss below, those with more animus towards a group will also tend to have incorrect beliefs that members of that group commit more crimes.

2.1 Full Information Policing

If the officer knows the true crime rates p_A and p_B (and his own t_A and t_B parameters), maximizing his utility function is straightforward. He may choose to allocate as much time as possible to policing one group or the other (i.e., $w_A = \underline{w}$ or $w_A = \overline{w}$), or an amount that is between these extremes, i.e. an interior allocation. Taking the derivative of u with respect to w_A and setting this equal to zero gives a unique candidate for an interior allocation:

$$w_A^\dagger(r_t, r_p) = \frac{r_t^2 r_p}{1 + r_t^2 r_p} \tag{2}$$

where we have reduced notation by defining:

$$r_t = \frac{t_A}{t_B} \quad \text{and} \quad r_p = \frac{p_A}{p_B}$$

These two summary parameters correspond exactly to the standard explanations for discrimination. The r_t parameter reflects the relative preferences for catching crimes among each group, which captures the possibility of taste-based discrimination. Given our definition above, an officer with $r_t > 1$ has animus towards group A, and $r_t < 1$ indicates animus towards group B. The r_p parameter reflects the *true* ratio in crime rates of the two groups, capturing the possibility of statistical discrimination. That is, if $r_p > 1$, the crime rate among members of group A is higher than the crime rate among members of group B, and if $r_p < 1$, the crime rate among members of group B is higher than the crime rate among members of group A. When it does not cause confusion, we suppress the r_t and r_p arguments when writing $w_A^\dagger$.

Since $r_t > 0$ and $r_p > 0$, it follows from (2) that $0 < w_A^\dagger < 1$. However, recall that the officer's choice must be between $\underline{w} > 0$ and $\overline{w} < 1$, so it is possible that the utility-maximizing policing choice is outside these bounds, which results in extreme policing, i.e. a "corner solution." In order to reduce the number of cases we need to consider in our analysis, we will assume that the parameters of the model are such that this does not occur:

Assumption 2. *If the officer has full information, then policing is non-extreme ($\underline{w} < w_A^\dagger < \overline{w}$).*

Given this assumption, an officer with full information will choose allocation $w_A^\dagger$, which we hereafter call the *full information policing*. This policing choice is increasing in r_t, meaning officers who have more animus towards group A will police this group more intensely. It is also increasing in r_p, meaning officers will spend more time policing a group when they believe crime is more prevalent among members of that group. Note that with full information policing, this belief will be based on *actual* crime rates. However, in our main analysis below, this belief may be distorted away from actual crime rates.

While the full information benchmark policing choice is "optimal" given the specified utility function for the officer, we should be clear that it is almost certainly not optimal for the policed communities, or society in general. As an extreme example, it could be the case that crime is more prevalent among members of group A, but that the officer has such strong animus towards group B that group B ends up being policed much more heavily.

Whenever the officer polices one group more than the other group, there is a *policing disparity*, given by:

$$\Delta^\dagger \equiv |w_A^\dagger - 1/2|.$$

Since r_t and r_p are how the model captures taste-based and statistical discrimination, it is worth discussing how $\Delta^\dagger$ can be decomposed into its two component parts. Formally, define $w_A^{\text{stat}} = w_A^\dagger(1, r_p) = r_p/(1 + r_p)$ to be the "statistical policing" allocation, which reflects what an officer does if he has no animus toward either group but statistically discriminates based on differences in the (true) crime rates. Then, the extent of statistical discrimination is captured by $w_A^{\text{stat}} - 1/2$ and the extent of taste-based discrimination is captured by $w_A^{\dagger} - w_A^{\text{stat}}$. Taken together, the policing disparity under full information policing can be decomposed as follows:[10]

$$\Delta^{\dagger} = |\underbrace{(w_A^{\dagger} - w_A^{\text{stat}})}_{\substack{\text{taste-based}\\ \text{discrimination}}} + \underbrace{(w_A^{\text{stat}} - 1/2)}_{\substack{\text{statistical}\\ \text{discrimination}}}| = |w_A^{\dagger} - 1/2|$$

While it should be uncontroversial that taste-based discrimination is normatively undesirable, the normative desirability of statistical discrimination is less clear. Focusing policing efforts toward communities with higher crime rates has the potential to make those communities safer. It may also increase community engagement if citizens do not perceive it to be too invasive (Lerman and Weaver 2014) and increase citizens' willingness to cooperate with police if community-oriented policing tactics are used (Peyton, Sierra-Arévalo, and Rand 2019). On the other hand, there is no guarantee that the "socially optimal" policing allocation corresponds to one in which an officer targets his efforts to higher crime communities. First, statistical discrimination can lead to inefficient stereotyping (see, for example, Coate and Loury 1993; Harcourt 2007; Glaser 2015; O'Flaherty and Sethi 2019). Second, "over-policing" certain social groups can have other spillovers, from reducing political participation (Weaver and Lerman 2010) to reinforcing those groups' status as "race-class subjugated" communities whose primary interaction with the state involves negative interactions with police (for an overview, see Soss and Weaver 2017).

2.2 Behavioral Policing

We now turn to our main analysis, which considers a situation in which the officer does not know the relative crime rates of the two groups (r_p), and forms this belief based on data generated

10. Note that taste-based and statistical discrimination may yield disparities against different groups. In this case, the policing disparity under full information will be closer to zero than the disparities generated by either effect independently.

by his policing choices. In reality, police departments collect data on crime from a wide variety of sources. For example, the department's crime statistics may include data about complaints, arrests and possibly even surveys of residents (such as the National Crime Victimization Survey). In this section, we assume that the data the officer in our model uses is entirely driven by the crime detected as a result of his policing choices. In Section 3, we explore the consequences of officers making choices based on data generated by *other* officers' choices as well.

A natural "inferential mistake" the officer might make is to assume that the amount of crime detected among members of each group reflects the crime rates of those groups, without accounting for the fact that one group may be more heavily policed than the other. As this involves the officer forming a posterior belief without conditioning on all relevant information, we call this *non-conditioning bias*.

We will allow the officer's non-conditioning bias to be more or less severe. Formally, we use the parameter $\nu \in [0, 1]$ to scale this severity. In the extreme where $\nu = 1$, he may simply compute the ratio of the number crimes detected among members of each group:

$$\tilde{r}_p(w_A, \nu = 1) = \frac{c_A}{c_B} = \frac{p_A w_A}{p_B(1 - w_A)} \tag{3}$$

Note that equation (3) is increasing in w_A, which implies that if the officer forms his belief about r_p in this fashion, he will think that there is a higher crime rate among members of group A (relative to group B) whenever that group is policed more heavily.

The correct way to form a belief about crime given the available data (c_A, c_B, w_A, and w_B) is to divide the crimes caught by the amount of time spent policing each group before making the comparison; i.e., to properly condition. When doing so (and with a large amount of data), the officer will form an accurate belief about the groups' crime rates. Formally, if $\nu = 0$ indicating that does not have any non-conditioning bias, then:

$$\tilde{r}_p(w_A, \nu = 0) = \frac{\frac{p_A w_A}{w_A}}{\frac{p_B(1-w_A)}{1-w_A}} = \frac{p_A}{p_B} = r_p \tag{4}$$

Expressions (3) and (4) respectively correspond to the two extreme situations: the officer makes the worst kind of inferential mistake and the officer does not make this inferential mistake at all. In general, we allow the officer to have a milder form of non-conditioning bias by introducing a parameter $\nu \in (0, 1)$, and the more general belief is:

$$\tilde{r}_p(w_A) = \frac{\frac{p_A w_A}{\nu+(1-\nu)w_A}}{\frac{p_B(1-w_A)}{\nu+(1-\nu)(1-w_A)}} = r_p \left(\frac{w_A(\nu + (1-\nu)(1-w_A))}{(1-w_A)(\nu + (1-\nu)w_A)} \right). \tag{5}$$

For conciseness, we will suppress the ν argument of $\tilde{r}_p$ in the remainder of the analysis. As ν approaches zero, the officer's belief about crime, $\tilde{r}_p$, becomes more accurate (i.e., approaches r_p). As ν approaches one, $\tilde{r}_p$ approaches the belief formed by the most extreme non-conditioning bias. More generally, as ν increases, the officer makes a more severe inferential mistake.

That police officers exhibit non-conditioning bias is not an outlandish proposition. Consider several examples. Using a case study of drug arrests in Oakland, California, Lum and Isaac (2016) demonstrates that data used in predictive policing algorithms perpetuates policing disparities since it is based on past policing patterns and does not appear to reflect *actual* drug use patterns. In her opinion in *Floyd v. New York*, U.S. District Judge Scheindlin writes "The City and its highest officials believe that blacks and Hispanics should be stopped at the same rate as their proportion of the local criminal suspect population" (p. 9). This is a prime example of non-conditioning bias, which is precisely what Judge Scheindlin finds troubling: "Instead, I conclude that the benchmark used by plaintiffs' expert—a combination of local population demographics and local crime rates (*to account for police deployment*) is the most sensible" (p. 9, emphasis added). Finally, Glaser (2015) recounts a particularly vivid example of non-conditioning bias when a former Los Angeles police chief told a reporter: "if officers are looking for criminal activity, they're going to look at the kind of people who are listed on crime reports" (p. 96). Of course, the "kinds of people who are listed on crime reports" will be disproportionately from highly policed communities and not necessarily representative of those who are prone to commit crimes.[11]

The steady state Thus far, we have described how the officer's belief depends on his policing decisions, given that he may have a non-conditioning bias that causes him to make an inferential mistake. We next consider how the officer's policing decisions depend on this potentially inaccurate belief. If the officer's policing decision and his (potentially inaccurate) belief are mutually reinforcing, then we will call this a "steady state" of the model since it serves as a useful prediction about what the officer would do.

Formally, the officer will choose the policing allocation w_A that is obtained by maximizing his utility given his belief $\tilde{r}_p(w_A)$ as defined in (5). This is his "best response" to his beliefs, which is given by:

$$w_A^{\text{br}}(r_t, \tilde{r}_p) = \begin{cases} \underline{w} & \text{if } \frac{r_t^2 \tilde{r}_p}{1+r_t^2 \tilde{r}_p} < \underline{w} \\ \frac{r_t^2 \tilde{r}_p}{1+r_t^2 \tilde{r}_p} & \text{if } \frac{r_t^2 \tilde{r}_p}{1+r_t^2 \tilde{r}_p} \in [\underline{w}, \overline{w}] \\ \overline{w} & \text{if } \frac{r_t^2 \tilde{r}_p}{1+r_t^2 \tilde{r}_p} > \overline{w} \end{cases} \tag{6}$$

Note that this best response resembles the full information case, but with the potentially incorrect belief $\tilde{r}_p$ replacing the true ratio r_p, and with the possibility of extreme policing (i.e., $\underline{w}$ or $\overline{w}$).

We have now characterized how his beliefs respond to his actions and how his actions respond to his beliefs. We now formally define what constitutes a solution to the model—a steady state.

11. While examples of non-conditioning bias abound, we make no specific claim about the severity of this bias across contexts. Individuals vary with their ability to make accurate inferences from data, and police departments use a variety of statistics, some of which may not be affected by non-conditioning bias. For this reason, we allow for relatively mild or severe forms of the bias, as represented by $\nu \in [0, 1]$.

Definition 3. *A **steady state of the unitary officer model** is a policing allocation w_A^* and a belief about crime rates $\tilde{r}_p^*$, where*

(i) w_A^* solves $w_A^* = w_A^{br}(r_t, \tilde{r}_p^*)$; and

(ii) $\tilde{r}_p^* = \tilde{r}_p(w_A^*)$.

Conveniently, such a steady state exists and is unique. That is, one of our main innovations is to demonstrate that it is possible to obtain a steady state solution of a model of policing in which an officer forms non-standard (and potentially inaccurate) beliefs. Put another way, even though more intense policing of one group creates a feedback channel, it is not the case that *any* policing allocation can be self-enforcing. To see why, note that there is a unique solution to $w_A = \frac{r_t^2 \tilde{r}_p(w_A)}{1+r_t^2 \tilde{r}_p(w_A)}$ given by:

$$\widehat{w}_A = w_A^{\dagger} + \frac{\nu(r_t^2 r_p - 1)}{(1-\nu)(1+r_t^2 r_p)} \tag{7}$$

If $\widehat{w}_A$ lies in $[\underline{w}, \overline{w}]$, then it corresponds to a steady state. Whenever $\widehat{w}_A$ does not lies in $[\underline{w}, \overline{w}]$, then there is a steady state involving extreme policing (i.e., a corner solution):

Proposition 1. *There is a unique steady state in which the officer chooses a policing allocation*

$$w_A^* = \begin{cases} \underline{w} & \text{if } \widehat{w}_A < \underline{w} \\ \widehat{w}_A & \text{if } \widehat{w}_A \in [\underline{w}, \overline{w}] \\ \overline{w} & \text{if } \widehat{w}_A > \overline{w} \end{cases}$$

and forms a (potentially inaccurate) belief $\tilde{r}_p^*$ using (5).

Proof All proofs are in the appendix.

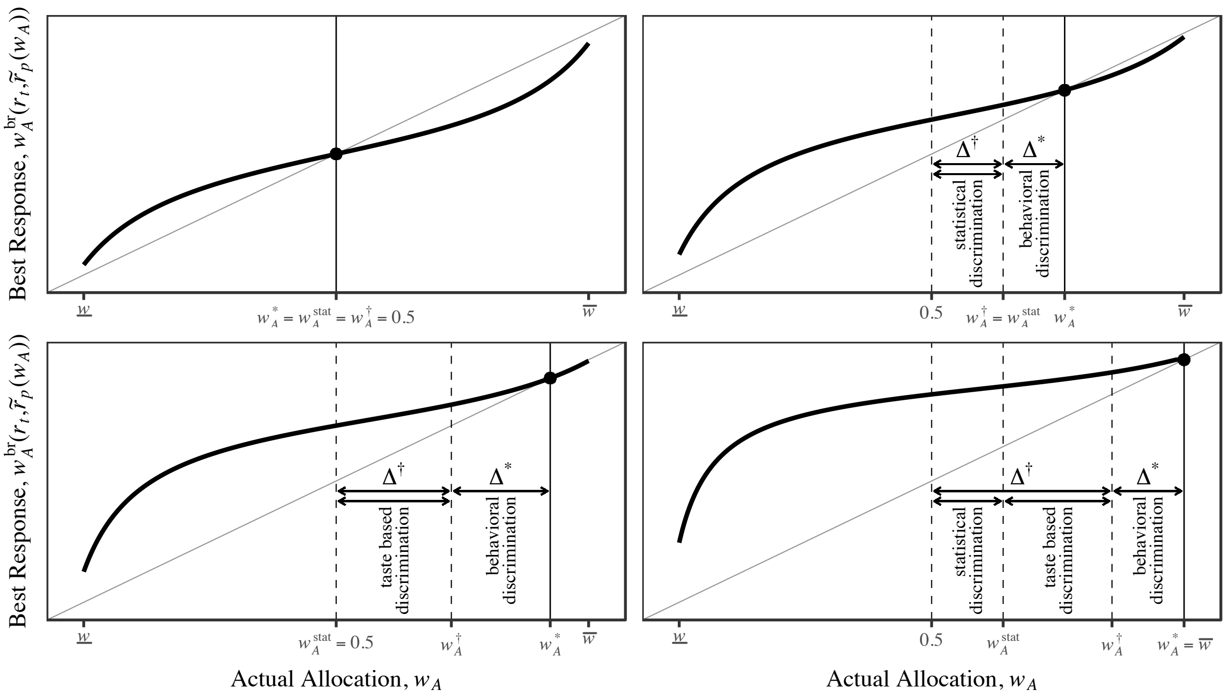
A natural way to conceptualize a steady state is in a dynamic setting. An officer chooses a policing allocation for some "time period," and then forms an updated belief about crime rates using (5) and given the data generated by his policing allocation. If the officer wants to change his policing allocation given this updated belief— i.e., $w_A^{\text{br}}(r_t, \tilde{r}_p(w_A)) \neq w_A$—then he is not in a steady state. If the officer wants to continue to use the same policing allocation given his updated belief—i.e., $w_A^{\text{br}}(r_t, \tilde{r}_p(w_A)) = w_A$—then he is in a steady state.

Figure 1 illustrates the steady state policing allocation in a way that corresponds to this dynamic process. In each panel (which vary in their values of r_t and r_p), the black curves trace out $w_A^{\text{br}}(r_t, \tilde{r}_p(w_A))$ as a function of w_A. The grey 45-degree line represents points where the best response allocation equals the actual allocation. Starting at any point w_A, if the $w_A^{\text{br}}(r_t, \tilde{r}_p(w_A))$ curve lies above the 45 degree line, then an officer who initially policies at allocation w_A will generate a belief about the relative crime rates that makes him want to police group A more. Conversely, if the curve lies below the 45 degree line, an officer starting at w_A would want to police group A less. A steady state is represented by an intersection of the black curve and the 45 degree line, where the officer would not want to change his allocation.

Setting aside the rest of the markings on the graph for the moment, note that in all but the bottom right panel the intersection is an interior allocation, which corresponds to a unique steady state. Before we explore how the officer's choice generates discrimination, we first note an important property of the officer's steady state policing allocation. As we show in Appendix D.2, the steady state allocation in Proposition 1 is "stable" in the sense that it is not sensitive to small perturbations.[12] Visually, this is because the best response curve lies below the 45 degree line above the steady state, and above the 45 degree line before the steady state. Roughly speaking, in the context of our model, this means that if the officer "accidentally" were to police one group a little more (or a little less) than their steady state allocation prescribes, the best response to his new belief (induced by the mistake) would be to move back towards the steady state.

12. In many models of sorting and statistical discrimination, there are multiple equilibria (or steady states), some of which are unstable (see, for example, Coate and Loury 1993; Benabou 1993; Chaudhuri and Sethi 2008). Unstable equilibria (or steady states) are undesirable because they only exist if the parameters of the model are exactly right. In the real world, people sometimes make small errors when making decisions, and so it is useful to know that an equilibrium (or steady state) will persist even when these small mistakes occur.

Figure 1: In each panel, we plot the officer's best response $w_A^{\text{br}}(r_t, \tilde{r}_p(w_A))$ as a function of his actual allocation of policing w_A. A steady state of the model occurs where $w_A^{\text{br}}(r_t, \tilde{r}_p(w_A))$ intersects the diagonal line—i.e., at a fixed point, denoted by a large dot. Each panel depicts steady states for different parameter values. We also depict the disparities caused by statistical, taste-based and behavioral discrimination in each the steady state. For the left panels, crime rates are equal and for the right panels, group A's crime rate is higher $r_p > 1$. For the top panels, the officer has no animus and for the bottom panels, the officer has animus against A, $r_t > 1$.



The difference between the panels in figure Figure 1 is that in the top panels the officer has no animus towards either group ($r_t = 1$), while in the bottom panels he has animus towards group A. In the left panels there is no difference in actual crime rates ($r_p = 1$), while in the right panels the true crime rate is higher in group A ($r_p > 1$).

Combining, the top left panel depicts a scenario with equal crime rates and no officer animus, $r_p = r_t = 1$. In this situation, despite making inferential mistakes, the officer's policing allocation is equal, $w_A^* = 1/2$. If the officer were to police group A more or less, there would be "self-correction" in the sense described above: he would move back towards the steady state with equal policing.

However, equal policing is fragile to changes in the exogenous parameters r_t and r_p. The bottom left panel demonstrates a situation with equal crime rates, but where the officer has animus toward group A. As the figure depicts, without making an inferential mistake, the officer's animus toward group A causes him to engage in taste-based discrimination against group A so that $w_A^\dagger > 1/2$ (and thus $\Delta^\dagger > 0$). However, his non-conditioning bias causes him to police group A even more than he would due to his animus alone, $w_A^* > w_A^\dagger$.

Formally, if the officer chooses a policing allocation w_A^* in a steady state, then we define the policing disparity relative to full information policing as:

$$\Delta^* \equiv |w_A^* - w_A^\dagger|.$$

This is the "excess disparity" caused by the fact that the officer makes an inferential mistake when forming his belief about the two crime rates. As a result, this excess disparity arises due to what we will call *behavioral discrimination*. We denote total discrimination as $\Delta = \Delta^\dagger + \Delta^*$. Returning to the bottom left panel of figure Figure 1, in this steady state about half of the officer's discrimination is driven by taste and about half is driven by non-conditioning bias.

Behavioral discrimination can also occur in the absence of officer animus. The top right panel indicates a case where $r_t = 1$ but $r_p > 1$. So, some excess policing of group A is explained by different crime rates (again $w_A^\dagger > 1/2$, and $\Delta^\dagger > 0$), but the officer believes these differences are bigger than they really are. As with the illustration of taste-based discrimination, this force roughly doubles the policing disparity relative to full information policing.

Finally, the bottom right panel shows a case where group A has a higher crime rate and the officer has animus towards this group. In this case, even though the full information policing allocation would be interior, with behavioral policing no matter what feasible allocation he chooses he would like to police group A even more, leading to extreme policing of this group. As demonstrated below, such extreme policing does not require both officer animus and differential crime rates, but will generically occur as long as the officer's non-conditioning bias is sufficiently strong.

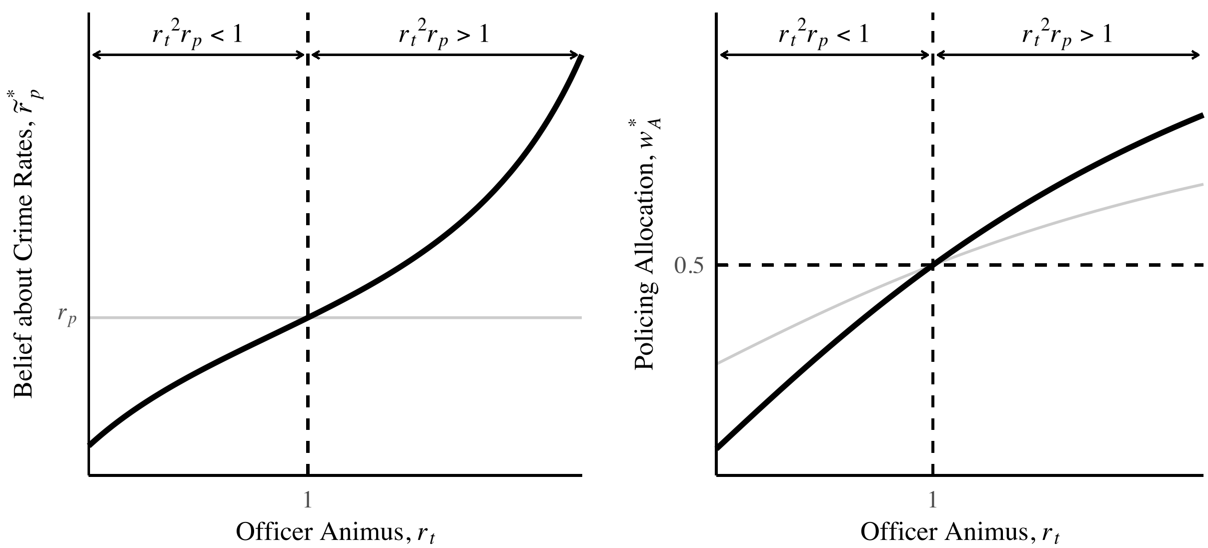
The officer's non-conditioning bias creates a link between taste-based and statistical discrimination. For an officer with any positive level of this bias, taste-based and statistical discrimination are no longer two mutually exclusive channels through which policing disparities emerge. In fact, what we term "behavioral discrimination" is formally equivalent to the excess statistical discrimination that is caused by exaggerated beliefs about relative crime rates. When conceptualized in this way, our model shows that taste-based discrimination can *cause* a (inaccurate) statistical discrimination. And since an officer's animus can cause distorted beliefs about crime rates, our model maps into an intuition in the academic literature (and in popular discourse) that the empirical phenomenon of prejudice will typically involve both racial animus and incorrect beliefs.

To be more concrete about how this works in our model, consider the following. First, the officer's animus causes him to allocate more policing effort toward one group. Then, since he spends more time policing that group, he sees more crimes among members of that group. Finally, his non-conditioning bias causes him to infer that the increased number of crimes he observes is an indication that the crime rate among members of that group is higher than it actually is. As a result (and notwithstanding his animus), his non-conditioning bias causes him to to *sincerely believe* that some (or even most) his overpolicing of one group is justified by the prevalence of crime among members of that group. As Gelman, Fagan, and Kiss (2007) point out: "Police often point to the high rates of seizures of contraband, weapons, and fugitives in such stops, and also to a reduction of crime, to justify such aggressive policing" (p. 814).

More generally, as the officer's animus increases, then his non-conditioning bias causes increasingly distorted beliefs and increasing levels of behavioral discrimination. In Figure 2, we plot both his steady state belief and his steady state policing allocation as a function of his animus. The solid grey lines indicates his steady state belief and allocation under full information policing (i.e., if he did not have a non-conditioning bias), and the solid black lines indicates his steady state belief and allocation when he has a non-conditioning bias.

Note that without non-conditioning bias, his belief does not depend on his animus and remains constant (at the true crime rate) no matter how much animus he has. In other words, taste-based and statistical discrimination are independent of one another with full information policing. However, once he has a non-conditioning bias, as r_t increases he forms increasingly exaggerated beliefs about the relative crime rate among members of group A. This causes his policing allocation to be even more unequal than it would under full information policing, as the right panel shows.

Figure 2: The solid grey lines depicts his policing allocation and belief about relative crime rates under full information policing, and the solid black lines depicts his policing allocation and belief about relative crime rates under behavioral policing. As the officer's animus toward group A increases, he polices group A more and forms an increasingly exaggerated (and incorrect) belief that crime is relatively more prevalent among members of group A.



While we have depicted how non-conditioning bias amplifies taste-based discrimination, it is also the case that it amplifies discrimination due solely to differences in crime rates. In other words, if crime rates are not equal between groups, non-conditioning bias causes even statistical discriminators (i.e., those with no animus) to overpolice one group as though they had animus toward that group.

The previous analysis suggests that behavioral discrimination will tend to amplify policing disparities caused by taste-based and/or statistical discrimination. We now explore exactly when and how this amplification occurs.

There is one situation where behavioral policing does not amplify policing disparities.[13] If there would be no policing disparity with full information policing, then behavioral policing cannot amplify policing disparities. Formally, this occurs if $r_t^2 r_p = 1$ so that $w_A^* = w_A^\dagger = 1/2$. In this case, behavioral policing does not *by itself* lead to discrimination against one group. One way that this may occur is if the officer has no animus towards either group ($r_t = 1$) and crime is group-invariant ($r_p = 1$). It is also possible if the officer has animus towards one group but the other group has a higher crime rate (high enough to exactly offset the officer's animus)—formally, this occurs if $r_p = 1/r_t^2$.

However, this is a very specific situation. If $r_t^2 r_p < 1$, then $w_A^* < w_A^\dagger$, meaning the officer polices group A less than he would with full information (and polices group B more). Conversely, when $r_t^2 r_p > 1$, the officer polices group A more than he would with full information (and polices group B less), $w_A^* > w_A^\dagger$. In both cases, $\Delta^* > 0$, meaning that behavioral policing generically amplifies whatever disparities would exist if officer formed accurate beliefs about crime rates.

Proposition 2. *For any $\nu \in (0, 1)$:*

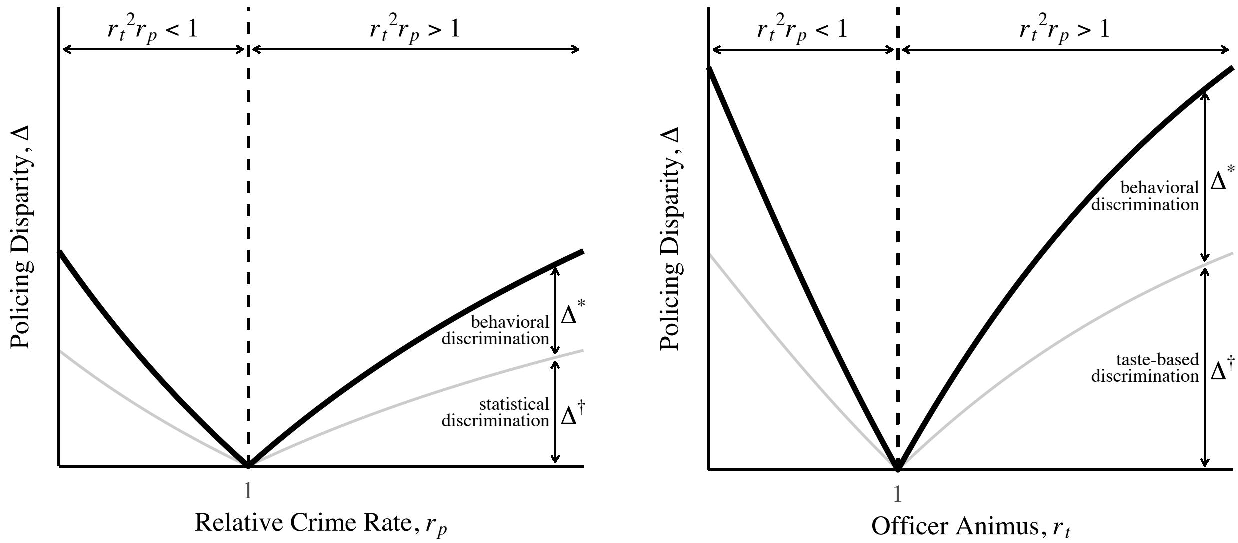
(i) If $r_t^2 r_p = 1$, then there is no policing disparity (since $w_A^* = w_A^\dagger = 1/2$), and the officer has correct beliefs about crime, $\tilde{r}_p^* = r_p$.

(ii) If $r_t^2 r_p \neq 1$, then behavioral policing amplifies existing disparities: $w_A^* > w_A^\dagger > 1/2$ if $r_t^2 r_p > 1$ and $w_A^* < w_A^\dagger < 1/2$ if $r_t^2 r_p < 1$ (alternatively, $\Delta^* > 0$), and the officer has incorrect beliefs, $\tilde{r}_p^* \neq r_p$.

If the officer's policing allocation is not extreme, then the disparity caused by behavioral discrimination is strictly positive as as $r_t^2 r_p$ moves away from 1. Figure 3 illustrates. In the left panel,

13. If we relax Assumption 2, then there is a second situation in which there is no behavioral discrimination. If the officer would engage in extreme policing regardless whether he has a non-conditioning bias, then trivially, behavioral policing does not increase the policing disparity. Formally, this occurs if $w_A^* = w_A^\dagger = \overline{w}$ or $w_A^* = w_A^\dagger = \underline{w}$. However, this scenario is qualitatively less interesting since it occurs solely due to the fact that the officer's choice is constrained to be in $[\underline{w}, \overline{w}]$ and there is no "room" for him to discriminate any more than he otherwise would with full information.

Figure 3: In each panel, we plot the policing disparity that emerges in a non-extreme steady state of the model, as a function of the true relative crime rate (left panel) and the officer's animus toward group A (right panel). As long as $r_t^2 r_p \neq 1$, the officer always engages in either statistical or taste-based discrimination, *as well as* behavioral discrimination.



we plot policing disparities as a function of the (true) relative crime rate, r_p. In the right panel, we plot policing disparities as a function of the officer's animus, r_t. In each panel, the grey line depicts the policing disparity caused by statistical and taste-based discrimination and the black line depicts the entire policing disparity. Note that in either panel, as long as $r_t^2 r_p \neq 1$, then behavioral discrimination causes the policing disparity to be higher than it otherwise would have been with only taste-based and statistical discrimination.

For Proposition 2 and Figure 3, we assume that ν is intermediate. As we demonstrate in the next result, as the severity of the officer's inferential mistake (as reflected by the value of ν) increases, the policing disparity increases until the officer reaches extreme policing. Strikingly, as long as behavioral policing causes any *any* disparity, then as the officer's inferential mistake become more severe, it eventually leads him to engage in extreme policing.

Proposition 3. *Assume* $r_t^2 r_p \neq 1$. *Then:*

(i) When the officer does not engage in extreme policing ($\underline{w} < w_A^* < \overline{w}$)*, the policing disparity*

caused by behavioral discrimination (Δ^*) is strictly increasing in ν.

(ii) There exists a $\widehat{\nu} < 1$ such that if $\nu \geq \widehat{\nu}$, the officer engages in extreme policing of one group and the policing disparity is at its maximum, i.e. $w_A^* = \underline{w}$ or $w_A^* = \overline{w}$.

Figure 4: As the severity of the officer's inferential mistake (i.e., ν) increases, so do policing disparities caused by behavioral discrimination. Moreover, very severe non-conditioning biases can cause such severe behavioral discrimination that the officer engages in extreme policing when he would not do so under full information policing.

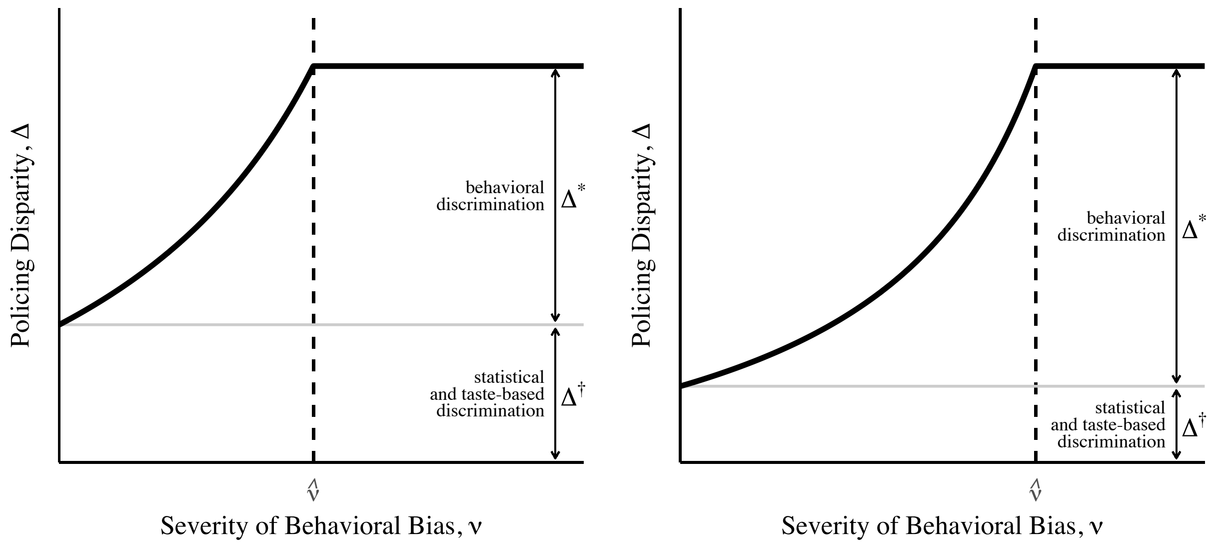


Figure 4 depicts the result for two situations. In each panel, the x-axis plots ν and the y-axis plots the policing disparity. The left panel depicts a situation where statistical and taste-based discrimination cause a large disparity, and the right panel depicts a situation where statistical and taste-based discrimination cause a small disparity. In either case, at any interior allocation behavioral discrimination strictly increases as ν increases, and if ν is sufficiently high the officer will engage in extreme policing.

In this section, we have analyzed a model of policing by a representative officer. With our analysis, we make three main points. First, we demonstrate that there is a policing allocation in which an officer makes an inferential mistake that affects their policing, and where their policing decision feeds back into and reinforces their (incorrect) beliefs. If the officer makes an inferential mistake when making decisions about how to allocate policing resources, we call this *behavioral policing*.

Second, behavioral policing generically causes the officer to engage in behavioral discrimination in which he overpolices one group because he forms an exaggerated belief that the relative crime rate among members of that group is higher than it actually is. An implication of this is that taste-based discrimination can perversely cause (inaccurate) statistical discrimination. Recall we previously noted that use of the term "prejudice" in prior work and popular discourse often conflates preferences and beliefs. In a sense, our results provide a justification for conflating these two mechanisms, as they will inevitably go together for those who make the kind of inferential mistake we study. However, the link between preferences and beliefs requires people to process information incorrectly. So, if reducing animus itself is not possible, it may still be possible to reduce prejudice by training officers to engage in more accurate statistical reasoning about why they may be observing cross-group differences in crime (or other relevant data).

Third, we showed that behavioral discrimination amplifies the policing disparity caused by taste-based and/or statistical discrimination. In particular, if $\nu > 0$ and $r_t^2 r_p \neq 1$, then he will always overpolice one group more than he otherwise would with full information. Moreover, the extent to which behavioral discrimination amplifies existing a policing disparity increases with the severity of his inferential mistake. As a result, behavioral policing can sometimes dramatically increase policing disparities, even inducing the officer to engage in extreme policing of one group.

Our model provides new insights about the causes of policing disparities. However, one potential limitation is that the data collected from the officer's own policing decisions (i.e. w_A^*) is the only thing causing him to form distorted beliefs. In reality, police departments are comprised of a multiple police officers with diverse preferences, and all of their individual policing choices end up contributing to the department's overall assessment of crime across communities. In the next section, we extend the model to look at how the presence of multiple, heterogeneous police officers affects our findings. Strikingly, we show that one officer's animus can "spill over" to a second officer, distorting that second officer's belief about crime rates and causing them to discriminate.

3 Model with Multiple, Heterogeneous Officers

To study how the dynamics of the model are different with multiple decision-makers, we analyze the simplest such environment: with two officers, indexed by $i \in \{1, 2\}$. Both officers choose how much time to allocate to group A, $w_{A,i} \in [\underline{w}, \overline{w}]$, with the remainder allocated to group B: $w_{B,i} = 1 - w_{A,i}$. In this section, let $w_J = w_{J,1} + w_{J,2}$ represent the *total* policing of group J. (Note that in this section, $2\underline{w} \leq w_J \leq 2\overline{w}$, since there are two officers each allocating 1 unit of time.) Let $c_{J,i} = p_J w_{J,i}$ be the number of crimes caught among group J by officer i, and $c_J = p_J w_J$ the total crime caught among members of group J.

We assume each officer cares only about the number of crimes that he catches:

$$u_i(c_{A,i}, c_{B,i}) = t_{A,i}\sqrt{c_{A,i}} + t_{B,i}\sqrt{c_{B,i}} = t_{A,i}\sqrt{p_A w_{A,i}} + t_{B,i}\sqrt{p_B(1 - w_{A,i})} \tag{8}$$

This utility function allows us to isolate the affect of distorted beliefs on policing since it means that there is no *direct* effect of officer j's behavior on the utility of officer i. There will only be an *indirect* effect of the other officer's behavior via officer i's belief. If instead each officer's utility were to be defined over the total crime caught, then the policing behavior of the other officer has a direct effect on his own best response, and we would not be able to cleanly isolate how much distorted beliefs affect policing decisions.

By an identical analysis to the case of the single officer with full information about the crime rates and assuming a non-extreme policing, the best response for each officer i depends on his animus ($r_{t,i}$) and the true ratio of crime rates (r_p):

$$w^{\dagger}_{A,i} = \frac{r^2_{t,i} r_p}{1 + r^2_{t,i} r_p}. \tag{9}$$

Because each officer's utility only depends on the crimes he catches, this allocation does not depend on the beliefs or behavior of the other officer in any way.

We also define the officers' beliefs in a similar way to the single officer model, but accounting for the fact that there are now two officers making policing allocations:

$$\tilde{r}_{p,i}(w_A) = \frac{\frac{c_A}{\nu_i + (1-\nu_i)w_{A,1} + \nu_i + (1-\nu_i)w_{A,2}}}{\frac{c_B}{\nu_i + (1-\nu_i)w_{B,1} + \nu_i + (1-\nu_i)w_{B,2}}} = \frac{\frac{c_A}{2\nu_i + (1-\nu_i)w_A}}{\frac{c_B}{2\nu_i + (1-\nu_i)(2-w_A)}} \tag{10}$$

Note that each officer's belief in the multiple officer model is indexed by i since each officer can, in principle, differ with respect to the severity of their non-conditioning bias (i.e., have different values of ν_i).

This definition implicitly assumes that each officer's failure to correct for policing intensity is symmetric in the sense that they fail to adjust for both their choice and the other officer's choice. In Appendix D.3, we briefly consider an alternative version of this bias where the officers adjust differently for their own behavior and the other officer's behavior. The key property of the symmetric version we study here, as well as the version we study in the appendix, is that officer i's belief about the relative prevalence of crime among members of group A is increasing in how much the *other* officer polices group A.

Each officer's best response also resembles the single officer case:

$$w^{\text{br}}_A(r_{t,i}, \tilde{r}_{p,i}) = \begin{cases} \underline{w} & \text{if } \frac{r^2_{t,i}\tilde{r}_{p,i}}{1+r^2_{t,i}\tilde{r}_{p,i}} < \underline{w} \\ \frac{r^2_{t,i}\tilde{r}_{p,i}}{1+r^2_{t,i}\tilde{r}_{p,i}} & \text{if } \frac{r^2_{t,i}\tilde{r}_{p,i}}{1+r^2_{t,i}\tilde{r}_{p,i}} \in [\underline{w}, \overline{w}] \\ \overline{w} & \text{if } \frac{r^2_{t,i}\tilde{r}_{p,i}}{1+r^2_{t,i}\tilde{r}_{p,i}} > \overline{w} \end{cases}$$

which is increasing in both his animus towards group A and his belief about the relative prevalence of crime among members of group A. This observation, combined with the fact that each officer's belief is affected by the policing allocation of the other officer, gives the intuition for the main result in this section. First, as officer 1's animus toward group A increases, this leads him to police group A more heavily (and, as in the previous section, this effect is amplified by inaccurate belief formation). And second, as long as officer 2 does not account for officer 1's animus-driven increased policing of group A, it will also lead officer 2 to believe (inaccurately) that group A has a higher crime rate. As a result, the animus of one officer ends up spilling over into the behavior of the other officer.

In the remainder of this section, we first demonstrate that this property holds in a steady state of the model, and then explore some more subtle properties of the resulting policing allocations and beliefs about crime. Formally, we define a solution of the multiple officer model as follows:

Definition 4. *A* ***steady state of the model with two officers*** *is a pair of allocation choices* $(w^*_{A,1}, w^*_{A,2})$ *and vector of beliefs* $(\tilde{r}^*_{p,1}, \tilde{r}^*_{p,2})$ *such that for all* $i \in \{1, 2\}$*:*

(i) $w^*_{A,i} = w^{br}_A(r_{t,i}, \tilde{r}^*_{p,i})$*, and*

(ii) $\tilde{r}^*_{p,i} = \tilde{r}_{p,i}(w^*_A)$ *is given by equation (17) evaluated at* w^*_A*.*

(The definition naturally extends to more than two officers.)

With multiple officers, it is difficult to obtain closed-form solutions. However, it is straightforward to show that a steady state exists, and in any steady state that meets a stability condition analogous to the single-officer model (see appendix B.2), the comparative statics are consistent with the conjectures above. We are primarily interested in the role that distorted beliefs play in policing disparities. More specifically, we examine how discrimination can "spill over" from one officer to another.

Our main result in the multiple officer model illustrates how this occurs:

Proposition 4. *In the model with two officers, a steady state exists. At any stable interior steady state allocation:*

(i) Each officer's allocation to group A ($w^*_{A,i}$) is strictly increasing in the animus of either officer, $r_{t,1}$ or $r_{t,2}$, and

(ii) If the officers collectively spend more than half of their time policing group J ($w^*_J > 1$), then each officer's allocation to group J is strictly increasing in the non-conditioning bias of either officer, ν_1 or ν_2.

In words, part (i) states that as either officer has more animus towards a group, *both* officers end up policing that group more. This is because the other officer (whose animus remains unchanged) does not fully correct for how his peer's increased policing of the group inflates the number of crimes caught among members of that group. In this sense, taste-based discrimination is contagious across officers.

Part (ii) states that whenever the officers collectively spend more time policing one group than the other, increasing the non-conditioning bias of either officer makes both officers decide to police that group even more. The intuition comes from the fact that whenever one group is policed more than the other, increasing one officer's non-conditoning bias has a direct effect on how much he polices that group (increasing it), and then spills over into the other officer's behavior. In this sense, inferential mistakes are contagious across officers.

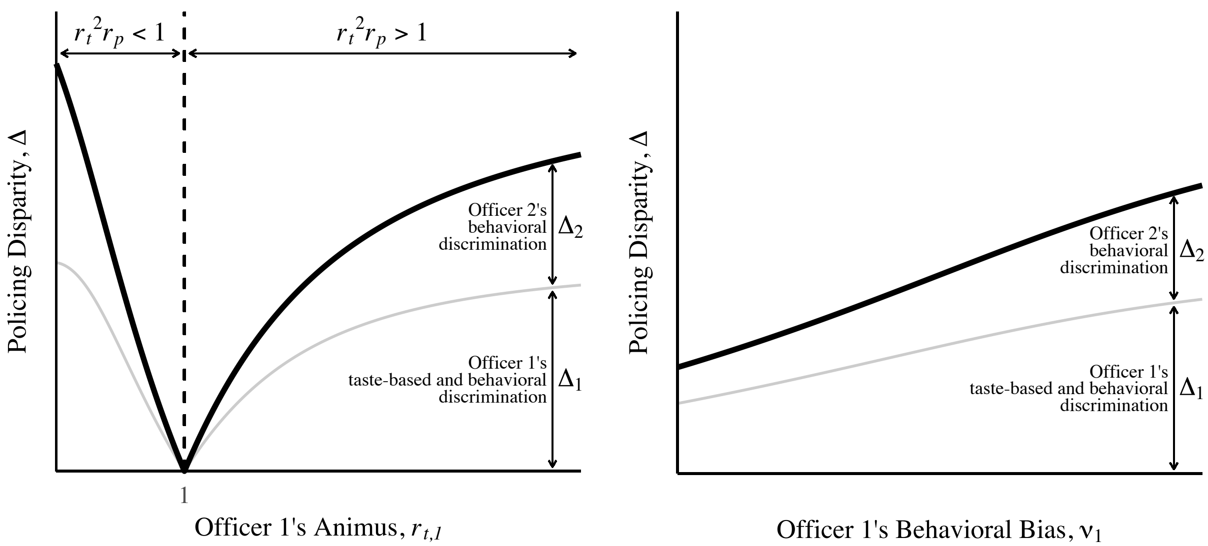
Finally, we illustrate how this affects individual and aggregate discrimination. To consider each officer's discrimination separately, define the policing disparities generated by each officer as follows:

$$\Delta_i^\dagger = |w^\dagger_{A,i} - 1/2| \qquad\qquad \Delta_i^* = |w^*_{A,i} - w^\dagger_{A,i}|$$

We also define the officer-level discrimination as $\Delta_i = \Delta_i^\dagger + \Delta_i^*$, and total discrimination by both officers as $\Delta = \Delta_1 + \Delta_2$.

In Figure 5, we depict examples of contagious prejudice (left panel) and contagious inferential mistakes (right panel). For both panels, we assume that crime rates are equal ($r_p = 1$), and that officer 2 has no animus toward either group (so that $r_{t,2} = 1$). The grey dashed line indicates officer 1's policing disparity, which is caused by both taste-based and behavioral discrimination. The black line depicts the total disparity that arises from both officers' policing allocations. The gap between the grey dashed line and the black line gives the disparity caused by officer 2's behavioral discrimination.

Figure 5: When there are multiple officers, discrimination due to animus and inferential mistakes are contagious. In the left panel, we depict how officer 2 discriminates more as officer 1 has more animus. In the right panel, we depict how officer 2 discriminates more as officer 1's non-conditioning bias becomes more severe.



First consider the left plot, which illustrates thesee quantities as a function of $r_{t,1}$. Since officer 2 has no animus and crime rates are equal, he would not discriminate under full information policing even if officer 1 does. However, since officer 2 has a non-conditioning bias, he ends up discriminating *because of* officer 1's animus toward one of the groups. Moreover, the more animus that officer 1 has, the more officer 2 discriminates. Next, consider the right plot, where the x-axis represents ν_1. As officer 1 makes increasingly severe inferential mistakes (holding fixed the officer

2's non-conditioning bias at $\nu_2 = 1/2$), he polices group A more, as he is not accounting for the fact that the higher crime rate among this group is driven by his own animus. This *also* leads officer 2 to police group A more, since he has a non-conditioning bias, $\nu_2 > 0$. In the examples illustrated by both panels, officer 1's discrimination is contagious.

These findings suggest that efforts to reduce policing disparities by reducing officer animus (via training), or diversifying police forces to reduce the number of officers with animus, may be of limited effectiveness as long as some officers still have animus toward one or more groups. Given their non-conditioning bias, a bad apple (or even a well-intentioned, but naive apple) can both spoil the bunch.

4 Conclusion

In this paper, we have provided a unified behavioral theory of discrimination in policing. Our theory is *unified* because it allows for both group-based animus and statistical differences between groups. It is *behavioral* because it relaxes the standard (and unrealistic) assumption that decision makers must be fully Bayesian. In particular, we assume that police officials form beliefs about the relative prevalence of crime among members of two groups without fully accounting for the intensity with which they police each of those two groups. We call this failure to account for policing intensity *non-conditioning bias*.

We show that an officer with this kind of non-conditioning bias will generically overpolice one of two groups due to the fact that he forms exaggerated beliefs about the relative crime rate among members of that group. We call this *behavioral discrimination*. This kind of discrimination will amplify existing disparities caused by taste-based and/or statistical discrimination. Moreover, when an officer has a non-conditioning bias, then it no longer makes sense to treat taste-based and statistical discrimination as separate and independent channels through by which discrimination occurs. Indeed, behavioral discrimination is a form of *inaccurate* statistical discrimination. Our model thus shows how racial animus and discrimination based on incorrect beliefs are intertwined.

We also extend the model to examine how the behavioral policing by multiple officers can generate discrimination. Due to their non-conditioning biases, the group-based animus of one officer can “spill over” and affect the policing decisions of another officer who has no animus toward either group. The analysis suggests that even if a very small number of officers harbor animus and discriminate against one group, other officers may discriminate against that group too.

The mechanism by which our model produces discrimination also potentially sheds light on the source of political and social conflict over biased policing. Many police officials and policing advocates vehemently assert that policing disparities are justified (for many examples, see Gelman, Fagan, and Kiss 2007), while many activists and community leaders protest practices they view as racially discriminatory. In our theory of behavioral policing, officers form inaccurate (and often exaggerated) beliefs about the relative prevalence of crime among members of specific social groups. These beliefs can induce them to think that their disparate treatment of certain groups of citizens is justified by differences in crime rates, even when it is not.

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Supporting Information

"A Behavioral Theory of Discrimination in Policing"

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A Racial Profiling and the Geography of Policing

The analysis in the main text assumes that police officers decide to allocate their time between policing two groups of people. In the United States, the prevailing law is unclear about whether such group-based profiling is permissible (for an extended discussion, see Knowles, Persico, and Todd 2001). However, under the U.S. Constitution, policies that explicitly treat members of protected categories differently are subject to strict scrutiny (see *Brown v. Board of Education of Topeka*, 1954). A policy of explicitly using group membership to allocate policing resources is not likely to survive a strict scrutiny legal analysis.

We focus on this simple, but potentially illegal, decision-making process in text because it allows us to more clearly focus on our core arguments. However, it can be microfounded with a more complex model where a police chief decides how many policing resources to devote to two neighborhoods: 1 and 2. Formally, assume he devotes n_1 of his time to policing neighborhood 1 and $n_2 = 1 - n_1$ of his time to policing neighborhood 2. Also assume that each neighborhood is comprised of members of the two groups, A and B. Within a neighborhood i, we assume that police interact with a member of group A with probability α_i and a member of group B with probability $1 - \alpha_i$. If police encounters with residents are random and iid, then one way to interpret α_i is that it represents the proportion of neighborhood i that is comprised of members of group A. However, our flexible specification allows for the possibility that police come into contact with members of one group at a rate disproportionate to that group's share of the local population. (Although note that if α_i does not reflect the demographic makeup of neighborhood i, then we simply reintroduce concerns about racial profiling that motivate this microfoundation, just at a different point in the analysis.)

Conditional on a choice about how intensely to police each neighborhood, the share of group A individuals the police encounters is $\eta_A = n_1\alpha_1 + (1 - n_1)\alpha_2 = \alpha_2 + (\alpha_1 - \alpha_2)n_1$ and the share of group B individuals the police encounters is $\eta_B = n_1(1 - \alpha_1) + (1 - n_1)(1 - \alpha_2) = 1 - \eta_A$. Recall from the main text that w_A is defined as the share of time that the police officer devotes to policing group A, and $w_B = 1 - w_A$ is the corresponding share of time that the police officer devotes to policing group B. Then, η_A is equivalent to w_A and η_B is equivalent to w_B, and n_1 is a perfect proxy for w_A. More specifically, if police come into contact with group A more than group B in neighborhood 1 (alt. neighborhood 2), $\alpha_1 > \alpha_2$ (alt. $\alpha_1 < \alpha_2$), then increasing n_1 (alt. n_2) linearly increases w_A. Notice that in the extreme cases where $n_1 = 0$ and $n_1 = 1$, then $\eta_A = \alpha_2$ and $\eta_A = \alpha_1$, respectively. Then, α_1 and α_2 correspond the maximum and minimum possible allocations: $\underline{w} = \min\{\alpha_1, \alpha_2\}$ and $\overline{w} = \max\{\alpha_1, \alpha_2\}$.

In a model where police choose n_1 (and not w_A), the analysis in the main text is identical after substituting $\eta_A = \alpha_2 + (\alpha_1 - \alpha_2)n_1$ for w_A.

B Stability Conditions

The main text makes informal references to a stability condition in the single-officer model (which, as we will prove, always holds at the unique steady state for this version). More consequentially, Proposition 4 makes reference to a stability condition which, while standard, is not yet defined. We first discuss the condition in the simpler single-officer setting and then the extension to two officers.

B.1 Single Officer

The main intuition for the single officer steady state to be stable is as follows. Let w_A^* be a steady state allocation. If it were the case that $w_A^{\text{br}}(r_t, \tilde{r}_p(w_A^* + \epsilon)) > w_A^* + \epsilon$ for some small $\epsilon > 0$, then a small upward perturbation in the policing of group A would lead the officer to want to police this group even more. Conversely, if $w_A^{\text{br}}(r_t, \tilde{r}_p(w_A^* - \epsilon)) < w_A^* - \epsilon$ for some small $\epsilon > 0$, a downward perturbation would lead the officer to police group A even less. We would like a stability condition which ensures that neither happens: i.e., small deviations would "return" to the steady state when iterating the best response function.

As $\epsilon \to 0$, since w_A^{br} is continuous, a formal statement of the stability requirement is:

Definition 5. *Let $F(w_A) = w_A^{br}(r_t, \tilde{r}_p(w_A)) - w_A$. A steady state allocation $w_A^* \in [\underline{w}, \overline{w}]$ is stable if and only if* $\left.\frac{\partial F}{\partial w_A}\right|_{w_A = w_A^*} < 0$.

To tie this to the geometric discussion surrounding Figure 1, this is equivalent to $\left.\frac{\partial w_A^{\text{br}}}{\partial w_A}\right|_{w_A = w_A^*} < 1$, i.e, the slope of the optimal allocation curve is less than that of the 45 degree line where they intersect.

Recall that w_A^{br} may not be differentiable at points where it switches from a corner to an interior solution. However, this does not pose any problems for this definition. If there is a corner steady state where $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w} + \epsilon)) = \underline{w}$, for some $\epsilon > 0$, then the derivative of the best response at $\underline{w}$ is zero, and hence the steady state is stable. Similarly, $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w} - \epsilon)) = \overline{w}$, the the corner solution at $\overline{w}$ is stable.

If there is a corner steady state at $\underline{w}$ but $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w} + \epsilon)) > \underline{w}$ for any $\epsilon > 0$, then the right derivative of w_A^{br} is well-defined at $\underline{w}$ and the stability condition is defined with respect to this derivative (which may or may not hold). Similarly, if there is a corner steady state at $\overline{w}$ but $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w} + \epsilon)) < \overline{w}$ for any $\epsilon > 0$, the stability condition can be defined with respect to the well-defined left-derivative of w_A^{br} at $\overline{w}$.

B.2 Multiple Officers

The steady state condition for the two officer model can be written:

$$F_1(w_{A,1}, w_{A,2}) \equiv w_A^{\text{br}}(r_{t,1}, \tilde{r}_{p,1}(w_{A,1}, w_{A,2}, \nu_1)) - w_{A,1} = 0$$
$$F_2(w_{A,1}, w_{A,2}) \equiv w_A^{\text{br}}(r_{t,2}, \tilde{r}_{p,2}(w_{A,1}, w_{A,2}, \nu_2)) - w_{A,2} = 0$$

Now we put a little more structure on what it means for a steady state to be stable. More specifically, close to a steady state, we want that for any "small" perturbation to both players' strategies, if the officers iteratively choose best responses given their new beliefs, then the joint allocation would move back to the steady state. By standard results in the study of dynamic systems (e.g., Theorem 11.4 in Gintis 2009), this can be expressed by conditions on the matrix of the partial derivatives of the F_i functions:

Definition 6. *Let*

$$D(w_{A,1}, w_{A,2}) = \begin{bmatrix} \frac{\partial F_1}{\partial w_{A,1}} & \frac{\partial F_1}{\partial w_{A,2}} \\ \frac{\partial F_2}{\partial w_{A,1}} & \frac{\partial F_2}{\partial w_{A,2}} \end{bmatrix}.$$

A steady state in the two-officer model is is stable if:

(i) $trace(D(w^*_{A,1}, w^*_{A,2})) < 0$, *and*

(ii) $det(D(w^*_{A,1}, w^*_{A,2})) > 0$.

The first condition simplifies to

$$\left.\frac{\partial F_1}{\partial w_{A,1}}\right|_{w_A = w_A^*} + \left.\frac{\partial F_2}{\partial w_{A,1}}\right|_{w_A = w_A^*} < 0$$

Note that if both derivatives are negative (as required in the single officer model), this is always true.

The second condition becomes:

$$\left[\frac{\partial F_1}{\partial w_{A,1}}\frac{\partial F_2}{\partial w_{A,2}} - \frac{\partial F_1}{\partial w_{A,2}}\frac{\partial F_2}{\partial w_{A,1}}\right]_{w_A=w_A^*} > 0$$

To provide a more easily interpretable version of these conditions, define:

$$\begin{aligned} Y_i &= \left.\frac{\partial w_A^{\text{br}}}{\partial \tilde{r}_{p,i}}\right|_{\tilde{r}_{p,i}=\tilde{r}_{p,i}(w_{A,1}^*, w_{A,2}^*)} \\ Z_i &= \left.\frac{\partial \tilde{r}_{p,i}(w_{A,1}, w_{A,2})}{\partial w_{A,1}}\right|_{w_A=w_A^*} = \left.\frac{\partial \tilde{r}_{p,i}(w_{A,1}, w_{A,2})}{\partial w_{A,2}}\right|_{w_A=w_A^*}. \end{aligned}$$

Then:

$$\left.\frac{\partial F_i}{\partial w_{A,i}}\right|_{w_A=w_A^*} = (Y_i Z_i - 1) \qquad\qquad \left.\frac{\partial F_i}{\partial w_{A,-i}}\right|_{w_A=w_A^*} = Y_i Z_i$$

Plugging these into first stability condition gives:

$$(Y_1Z_1 - 1) + (Y_2Z_2 - 1) < 0 \iff Y_1Z_1 + Y_2Z_2 < 2 \tag{11}$$

and the second:

$$\begin{aligned} (Y_1Z_1 - 1)(Y_2Z_2 - 1) - (Y_1Z_1)(Y_2Z_2) > 0 \\ \iff Y_1Z_1 + Y_2Z_2 < 1 \end{aligned}$$

which is stronger than condition (11) and hence the binding constraint.

An intuition for this condition is that due to the complementarities between action and belief, the deviations that are most apt not to return to a steady state are those where both officers increase or both officers decrease their allocations. And $Y_1 Z_1 + Y_2 Z_2$ is the marginal change in the best response as *both* officers increase their allocation to group A. So, this condition states that if both officers were to allocate slightly more time to group A or both allocated slightly less, their best responses would move back toward the steady state allocation.

C Proofs

Proof of Proposition 1. Using Definition 3, a steady state policing allocation w_A solves

$$w_A^* = w_A^{\text{br}}(r_t, \tilde{r}_p^*(w_A^*))$$

At any interior solution, $w_A^{\text{br}}(r_t, r_p) = \frac{r_t^2 \tilde{r}_p(w_A)}{1+r_t^2 \tilde{r}_p(w_A)}$. Substituting (5) and solving this equation for w_A gives a unique solution $\widehat{w}_A$, defined by equation (7) in the main text. Thus when $\widehat{w}_A$ lies in $[\underline{w}, \overline{w}]$ it meets the condition for a unique steady state allocation, $w_A^* = \widehat{w}_A$.

To prove that the corner solutions lie where the proposition claims, it helps to first describe the shape of the function which in turn describes how the allocation would change if playing an unconstrained best response starting at w_A,

$$F(w_A) = \frac{r_t^2 \tilde{r}_p(w_A)}{1 + r_t^2 \tilde{r}_p(w_A)} - w_A,$$

on the full range of $[0, 1]$. This function is continuous and differentiable. It is immediate that $F(0) = 0$ and $F(1) = 0$,[14] and by the analysis above $F(\widehat{w}_A) = 0$. So, when $\widehat{w}_A \in (0, 1)$, there are three zeroes on $[0, 1]$, and when $\widehat{w}_A$ lies outside of this interval the only zeroes are at the endpoints

14. This implies that if we did not restrict the range to $[\underline{w}, \overline{w}]$, there would always be a steady state only policing either group, though this would not meet the stability condition whenever an interior steady state exists.

(and hence the function must be always positive or negative). Recall that:

$$\widehat{w}_A = \frac{r_t^2 r_p}{1 + r_t^2 r_p} + \frac{\nu(r_t^2 r_p - 1)}{(1-\nu)(1 + r_t^2 r_p)}$$

Rearranging and simplifying gives:

$$0 < \widehat{w}_A < 1 \iff \nu < r_t^2 r_p < 1/\nu$$

In order to see whether F is positive or negative as $w_A \to 0$ and $w_A \to 1$, we need to check F' at these two points. Taking the first derivative of F yields:

$$F'(w_A) = \frac{\nu r_p r_t^2 \left(2(1-\nu)w_A^2 - 2(1-\nu)w_A + 1\right)}{\left(\nu w_A^2 \left(r_p r_t^2 + 1\right) - 2\nu w_A + \nu + (1-w_A)w_A \left(r_p r_t^2 + 1\right)\right)^2} - 1$$

Evaluating at 0 and 1 gives:

$$F'(0) > 0 \iff r_t^2 r_p > \nu \qquad\qquad F'(1) > 0 \iff r_p r_t^2 < \frac{1}{\nu}$$

Since $\nu < 1/\nu$, there are three cases we must consider, corresponding to three possible shapes of the F function. In case (I), $r_t^2 r_p \geq 1/\nu$. When the inequality is strict, this implies F is increasing at 0, decreasing at 1, and has no interior root, and hence $F(w_A) > 0$ for $w_A \in (0,1)$. When $r_t^2 r_p = 1/\nu$, the only difference is that $F'(1) = 0$, but F is decreasing for w_A approaching 1, and this does not affect the rest of the argument. In case (II), $\nu < r_t r_p^2 < 1/\nu$, and so F is increasing at 0 and at 1, with an interior zero at $\widehat{w}_A$, and hence $F(w_A) > 0$ for $w_A \in (0, \widehat{w}_A)$ and $F(w_A) < 1$ for $w_A \in (\widehat{w}_A, 1)$. In case (III) $r_t^2 r_p \leq \nu$, and F is decreasing at 0 (or, in the case where $r_t^2 r_p = \nu$, flat at 0 but decreasing for small w_A), increasing at 1, and has no interior root, and hence $F(w_A) < 0$ for $w_A \in (0,1)$. Note that there can only be an interior steady state in case (II), and it must be the case that $F'(\widehat{w}_A) < 0$, i.e., the stability condition for this version of the model described in

Appendix B.1.

Now we can complete proving where the steady states lie and uniqueness by when the domain of the allocation choice is restricted to $[\underline{w}, \overline{w}]$. If $\widehat{w}_A \leq 0$ then the F function is in case (III) above, and so $F(\underline{w}) < 0$. If $0 < \widehat{w}_A < \underline{w}$, it is in case (II), but since $\underline{w} \in (\widehat{w}_A, 1)$ it must also be the case that $F(w_A) < 0$ for all $w_A \in [\underline{w}, \overline{w}]$. And returning to the definition of w_A^{br}, $F(\underline{w}) < 0$, implies $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w})) = \underline{w}$, meaning there is an extreme steady state at $\underline{w}$. $F(w_A) < 0$ also implies there is no interior steady state or steady state at $\overline{w}$ since $F(\overline{w}) < 0$, so this steady state is unique. If $\widehat{w}_A = \underline{w}$, then it is immediate that $F(\underline{w}) = \underline{w}$, and hence there is an extreme steady state at this bound, and this steady state is unique since $F(w_A) < 0$ for $w_A \in (\underline{w}, \overline{w}]$.

When $\underline{w} < \widehat{w}_A < \overline{w}$, $\widehat{w}_A$ is an interior steady state, and there can't be another interior state since there is no other point on $[\underline{w}, \overline{w}]$ where $F(w_A) = 0$. The F function is in case (II), which implies $F(\underline{w}) > 0$ and $F(\overline{w}) < 0$, so there is no steady state at the extremes. Thus the steady state is unique.

By a similar argument to the $\widehat{w}_A \leq \underline{w}$ case, if $\widehat{w}_A \geq \overline{w}$, then $w_A^{\text{br}}(r_t, \tilde{r}_p(\overline{w})) = \overline{w}$, and there can't be a steady state at $\underline{w}$ or on the interior. ∎

Proof of Proposition 2. Let $\nu \in (0, 1)$. Note that $\widehat{w}_A$ in the main text simplifies to

$$\widehat{w}_A = \frac{r_t^2 r_p - \nu}{(1-\nu)(1 + r_t^2 r_p)} \tag{12}$$

It follows that $\widehat{w}_A < 1/2$ for $r_t^2 r_p < 1$ and $\widehat{w}_A > 1/2$ for $r_t^2 r_p > 1$. To see this, note that using (12), $\widehat{w}_A < 1/2$ reduces to $r_t^2 r_p < 1$ and $\widehat{w}_A > 1/2$ reduces to $r_t^2 r_p > 1$.

We now consider the two cases in the statement.

Case 1. First suppose $r_t^2 r_p = 1$. Then, using the fact that $0 < \underline{w} \leq 1/2 \leq \overline{w} < 1$ (from Assumption 2), it is direct to see that $w_A^* = w_A^\dagger = 1/2$ for all $\nu \in (0, 1)$. Then, $\Delta^\dagger = \Delta^* = 0$. Moreover, since $w_A^* = 1/2$, using (5) it follows that the steady state belief is correct, $\tilde{r}_p^* = r_p$.

Case 2. Suppose $r_t^2 r_p \neq 1$. Under Assumption 2, $\underline{w} < w_A^\dagger < 1/2$ if $r_t^2 r_p < 1$ and $1/2 < w_A^\dagger <$

$\underline{w}$ if $r_t^2 r_p > 1$. It is immediate to see that $\Delta^\dagger = |w_A^\dagger - 1/2| > 0$.

Next, consider an interior steady state so that $w_A^* = \widehat{w}_A$. Since $0 < \nu < 1$, then it is immediate to see that $\widehat{w}_A \neq w_A^\dagger$ and therefore, $\Delta^* = |\widehat{w}_A - w_A^\dagger| > 0$.

We next consider a corner solution so that $w_A^* \in \{\overline{w}, \underline{w}\}$. If $r_t^2 r_p < 1$, then $w_A^* \neq \overline{w}$. To see this, assume by contradiction that $w_A^* = \overline{w}$. Then, since there is a corner solution, $\widehat{w}_A \geq \overline{w}$. We showed above that $\widehat{w}_A < 1/2$ if $r_t^2 r_p < 1$, which means that $\overline{w} < 1/2$. However, this contradicts our assumption that $\overline{w} \geq 1/2$ (from Assumption 1). By the same logic, if $r_t^2 r_p > 1$, then $w_A^* \neq \underline{w}$.

We then consider the only possible corner solutions. If $r_t^2 r_p < 1$, then a corner solution involves $w_A^* = \underline{w}$. Then, using Assumption 2, $w_A^* = \underline{w} < w_A^\dagger < 1/2$, and $\Delta^* = |w_A^* - w_A^\dagger| = w_A^\dagger - \underline{w} > 0$. If $r_t^2 r_p > 1$, then a corner solution involves $w_A^* = \overline{w}$. Then, using Assumption 2, $1/2 < w_A^\dagger < w_A^* = \overline{w}$, and $\Delta^* = |w_A^* - w_A^\dagger| = \overline{w} - w_A^\dagger > 0$.

Finally, note that if $w_A^* \neq 1/2$, then using (5) it follows that his steady state belief is incorrect, $\tilde{r}_p^* \neq r_p$. ∎

Proof of Proposition 3. Suppose that $r_t^2 r_p \neq 1$. First assume that $w_A^* = \widehat{w}_A$. Then,

$$\Delta^* = |\widehat{w}_A - w_A^\dagger| = \left| \frac{\nu(r_t^2 r_p - 1)}{(1-\nu)(1 + r_t^2 r_p)} \right|$$

Since $0 < \nu < 1$, this may be rewritten

$$\Delta^* = \frac{\nu}{1-\nu} \left| \frac{r_t^2 r_p - 1}{1 + r_t^2 r_p} \right|$$

Then, since $\frac{\nu}{1-\nu}$ is strictly increasing in ν, it follows that Δ^* is also strictly increasing in ν.

Next, assume that $w_A^* \in \{\underline{w}, \overline{w}\}$. We prove for $w_A^* = \underline{w}$, but the same logic applies for the other case. Then,

$$\Delta^* = |w_A^* - w_A^\dagger| = |\underline{w} - w_A^\dagger|$$

Since $\underline{w} < w_A^\dagger$ by Assumption 2, this may be rewritten

$$\Delta^* = w_A^\dagger - \underline{w} = \frac{r_t^2 r_p}{1 + r_t^2 r_p} - \underline{w}$$

Moreover, from the proof of Proposition 2, if $w_A^* = \underline{w}$, it must follow that $w_A^\dagger < 1/2$ Then, the total disparity is

$$\Delta^\dagger + \Delta^* = \left[\frac{1}{2} - \frac{r_t^2 r_p}{1 + r_t^2 r_p}\right] + \left[\frac{r_t^2 r_p}{1 + r_t^2 r_p} - \underline{w}\right] = 1/2 - \underline{w}.$$

Finally, we show that there exists a threshold $0 < \widehat{\nu} < 1$ such that $w_A^* \in \{\underline{w}, \overline{w}\}$ for $\nu \geq \widehat{\nu}$. First, by Assumption 2, $\widehat{w}_A \in (\underline{w}, \overline{w})$ for $\nu = 0$. Second, examining (7), as $\nu \to 1$, $\widehat{w}_A \to \infty$ for $r_t^2 r_p > 1$ and $\widehat{w}_A \to -\infty$ for $r_t^2 r_p < 1$. Since $0 < \underline{w} < w_A < \overline{w} < 1$, then for $\nu = 1$, $w_A^* \in \{\underline{w}, \overline{w}\}$. Finally, $\widehat{w}_A$ is continuous and increasing in ν for $r_t^2 r_p > 1$ and decreasing in ν for $r_t^2 r_p < 1$, so there exists some $\widehat{\nu} < 1$ such that for all $\nu \geq \widehat{\nu}$, $w_A^* \in \{\underline{w}, \overline{w}\}$. Then, from above, for all $\nu \geq \widehat{\nu}$, the policing disparity is at its maximum. ∎

Proof of Proposition 4. To prove the existence of a steady state allocation, define a function $G : [\underline{w}, \overline{w}]^2 \to [\underline{w}, \overline{w}]^2$ given by

$$G(w_{A,1}, w_{A,2}) \equiv (w_A^{\text{br}}(r_{t,1}, \tilde{r}_{p,1}(w_{A,1}, w_{A,1}), w_A^{\text{br}}(r_{t,2}, \tilde{r}_{p,2}(w_{A,1}, w_{A,1})).$$

This is a continuous mapping from a compact and convex set to itself, so by the Brouwer fixed point theorem there must be a $(w_{A,1}^*, w_{A,2}^*)$, such that $G(w_{A,1}^*, w_{A,2}^*) = (w_{A,1}^*, w_{A,2}^*)$, which is a steady state allocation, with corresponding steady state beliefs given by $\tilde{r}_{p,i}^* = \tilde{r}_{p,i}(w_{A,1}^*, w_{A,2}^*)$.

We now show the comparative static results. First, recall we can write the steady state conditions as the following system of equations:

$$F_1(w_{A,1}, w_{A,2}; r_{t,1}, \nu_1) = w_A^{\text{br}}(r_{t,1}, \tilde{r}_{p,1}(w_{A,1}, w_{A,2})) - w_{A,1} = 0$$
$$F_2(w_{A,1}, w_{A,2}; r_{t,1}, \nu_1) = w_A^{\text{br}}(r_{t,2}, \tilde{r}_{p,2}(w_{A,1}, w_{A,2})) - w_{A,2} = 0$$

For part (i), we prove the result as $r_{t,1}$ changes, but identical logic holds for $r_{t,2}$.

To implicitly differentiate the steady state conditions with respect to $r_{t,1}$, take the total derivative of F_1 and F_2 (at w_A^*, accounting for the fact that $w_{A,i}$ are a function of $r_{t,1}$:

$$\left.\frac{dF_1}{dr_{t,1}}\right|_{w_A=w_A^*} = \left(\left.\frac{\partial w_A^{\text{br}}}{\partial r_{t,1}}\right|_{w_A=w_A^*} + Y_1 \left(Z_1 \left.\frac{\partial w_{A,1}}{\partial r_{t,1}}\right|_{w_A=w_A^*} + Z_1 \left.\frac{\partial w_{A,2}}{\partial r_{t,1}}\right|_{w_A=w_A^*} \right) \right) - \left.\frac{\partial w_{A,1}}{\partial r_{t,1}}\right|_{w_A=w_A^*} = 0 \tag{13}$$

$$\left.\frac{dF_2}{dr_{t,1}}\right|_{w_A=w_A^*} = \left(Y_1 \left(Z_2 \left.\frac{\partial w_{A,1}}{\partial r_{t,1}}\right|_{w_A=w_A^*} + Z_2 \left.\frac{\partial w_{A,2}}{\partial r_{t,1}}\right|_{w_A=w_A^*} \right) \right) - \left.\frac{\partial w_{A,2}}{\partial r_{t,1}}\right|_{w_A=w_A^*} = 0 \tag{14}$$

where as in section B.2 we define:

$$Y_i = \left.\frac{\partial w_A^{\text{br}}}{\partial \tilde{r}_{p,i}}\right|_{\tilde{r}_{p,i}=\tilde{r}_{p,i}(w_{A,1}^*, w_{A,2}^*)}$$
$$Z_i = \left.\frac{\partial \tilde{r}_{p,i}(w_{A,1}, w_{A,2})}{\partial w_{A,1}}\right|_{w_A=w_A^*} = \left.\frac{\partial \tilde{r}_{p,i}(w_{A,1}, w_{A,2})}{\partial w_{A,2}}\right|_{w_A=w_A^*}.$$

Equations (13) and (14) are a system of two equations where we want to solve for $\frac{\partial w_{A,1}}{\partial r_{t,1}}$ and $\frac{\partial w_{A,1}}{\partial r_{t,1}}$. Define the following:

$$T_1 = \left.\frac{\partial w_{A,1}}{\partial r_{t,1}}\right|_{w_A=w_A^*} \qquad T_2 = \left.\frac{\partial w_{A,2}}{\partial r_{t,1}}\right|_{w_A=w_A^*} \qquad X = \left.\frac{\partial w_A^{\text{br}}}{\partial r_{t,1}}\right|_{w_A=w_A^*}$$

Then, we can rewrite this system of equations as

$$(X + Y_1 Z_1(T_1 + T_2)) - T_1 = 0$$
$$Y_1 Z_1(T_1 + T_2) - T_1 = 0$$

and goal is to solve for T_1 and T_2. This gives:

$$T_1 = X + \frac{XY_1Z_1}{1 - Y_1Z_1 - Y_2Z_2}$$
$$T_2 = \frac{XY_2Z_2}{1 - Y_1Z_1 - Y_2Z_2}.$$

Since we know that $X > 0$, $Y_i > 0$, and $Z_i > 0$, both of these are strictly positive if and only if $1 - Y_1Z_1 - Y_2Z_2 > 0$, which is exactly the stability condition for an interior steady state derived in section B.2. Finally, since $\Delta^* = |w^*_{A,i} - w^\dagger_{A,i}|$ and $w^\dagger_{A,i}$ is constant in $r_{t,1}$, then for each $i \in \{1, 2\}$, Δ^* increases in $r_{t,1}$.

For part (ii), we prove the result as ν_1 changes, but identical logic holds for ν_2. We now define the following:

$$N_1 = \left.\frac{\partial w_{A,1}}{\partial \nu_1}\right|_{w_A = w^*_A} \qquad N_2 = \left.\frac{\partial w_{A,2}}{\partial \nu_1}\right|_{w_A = w^*_A}$$

To implicitly differentiate the steady state conditions with respect to ν_1, take the total derivative of the equilibrium conditions at w^*_A, accounting for the fact that $w_{A,i}$ is a function of ν_1:

$$Y_1\left(Z_1N_1 + Z_1N_2 + \frac{\partial \tilde{r}_{p,1}}{\partial \nu_1}\right) - N_1 = 0 \qquad Y_2\left(Z_2N_1 + Z_2N_2\right) - N_1 = 0$$

Our goal is to solve for N_1 and N_2, which gives:

$$N_1 = \frac{\partial \tilde{r}_{p,1}}{\partial \nu_1}\left(\frac{Y_1(1 - Y_2Z_2)}{1 - Y_1Z_1 - Y_2Z_2}\right) \qquad N_2 = \frac{\partial \tilde{r}_{p,1}}{\partial \nu_1}\left(\frac{Y_1Y_2Z_2}{1 - Y_1Z_1 - Y_2Z_2}\right)$$

Again since we know that $X > 0$, $Y_i > 0$, and $Z_i > 0$, both of these are strictly positive at an interior steady state if and only if the stability condition is met and $\frac{\partial \tilde{r}_{p,1}}{\partial \nu_1} > 0$. This latter condition holds if $w_A = w_{A,1} + w_{A,2} > 1$ (i.e., group A receives a higher allocation than group B). Similarly, if $w_A < 1$, then $\frac{\partial \tilde{r}_{p,1}}{\partial \nu_1} < 0$ and hence both officers police group B more as ν_1 increases. ∎

D Generalizing the Model

D.1 More General Utility Function

A simple generalization is to write the utility function as:

$$u = t_A(w_A p_A)^\alpha + t_B(w_B p_B)^\alpha \tag{15}$$

where $\alpha \in (0, 1)$. So the formulation in the main text is the $\alpha = 1/2$ case. With $w_B = w - w_A$, the FOC is now:

$$\alpha t_A p_A (w_A p_A)^{\alpha-1} - \alpha p_B t_B((w - w_A)p_B)^{\alpha-1} = 0$$

Rearranging gives:

$$1 = r_t r_p^\alpha \left(\frac{w_A}{w - w_A}\right)^{\alpha-1}$$

Without even explicitly characterizing the solution, we can see that this is just a function of r_t and r_p (rather than the primitive p_J and t_J parameters) and the solution is increasing in r_t and r_p. Solving fully:

$$w_A = \left(\frac{\left(r_t r_p^\alpha\right)^{\alpha-1}}{1 + \left(r_t r_p^\alpha\right)^{\alpha-1}}\right) w$$

which is increasing in r_t and r_p as in the main formulation.

D.2 More General Best Response

A more general way to think about the officer utility is to make weaker assumptions, but directly built into the best response function. In particular, suppose that the officer still choose an allocation $w_A \in [\underline{w}, \overline{w}]$, where:

Assumption 3. *For any belief about the relative prevalence of crime among members of the two groups, $\tilde{r}_p \in \mathbb{R}_+$ (formed by equation 5), and relative animus parameter r_t, the officer has a unique best response allocation $w_A^{br}(r_t, \tilde{r}_p) \in [\underline{w}, \overline{w}]$ such that: (i) w_A^{br} is continuous and weakly increasing in both arguments, and (ii) where $w_A^{br} \in (\underline{w}, \overline{w})$, w_A^{br} is differentiable in both arguments and strictly increasing in r_t and $\tilde{r}_p$.*

Implicit in this definition is the fact that (1) the officer's animus can be captured by a single parameter r_t (rather than the primitive t_A and t_B in the main utility function) and that the best response is only a function of the ratio of the crime rates rather than the individual crime rates. Both are also true in the main model, as well as in the more general utility function given by equation (15).

Our main technical result in this section is that a stable steady state allocation always exists:

Proposition 5. *For any officer utility meeting Assumption 3, there exists a stable steady state allocation.*

Proof of Proposition 5. If $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w} + \epsilon)) = \underline{w}$ for some $\epsilon > 0$ or $w_A^{\text{br}}(r_t, \tilde{r}_p(\overline{w} - \epsilon)) = \overline{w}$ for some $\epsilon > 0$, then there is a stable corner steady state allocation. To complete the proof we need to show that if neither of these hold, there is an interior steady state. Let

$$F(w_A) = w_A^{\text{br}}(r_t, \tilde{r}_p(w_A)) - w_A$$

That is, $F(w_A)$ represents how he would change his allocation if starting from w_A, and a steady state is a point where $F(w_A^*) = 0$. If there is no stable corner solution, then it must be the case that $w_A^{\text{br}}(r_t, \tilde{r}_p(\underline{w} + \underline{\epsilon})) > \underline{w}$ for some small $\underline{\epsilon} \in (0, 1/2)$, and hence $F(\underline{w} + \underline{\epsilon}) > 0$. There must also be a $\overline{\epsilon} \in (0, 1/2)$ such that and $w_A^{\text{br}}(r_t, \tilde{r}_p(\overline{w} - \overline{\epsilon})) > 0$ and similarly $F(\overline{w} - \overline{\epsilon}) < 0$. By the continuity of w_A^{br} in $\tilde{r}_p$ and the continuity of $\tilde{r}_p$ in w_A, F is continuous in w_A, and so the intermediate value theorem implies there must be a $w_A^* \in (\underline{\epsilon}, \overline{\epsilon})$ such that $F(w_A^*) = 0$, where $F'(w_A) < 0$. Finally, since $F'(w_A) = \frac{\partial w_A^{\text{br}}}{\partial w_A} - 1$, then $F'(w_A^*) < 0 \iff \frac{\partial w_A^{\text{br}}}{\partial w_A}\Big|_{w_A = w_A^*} < 1$, and w_A^* is stable. ∎

There is no guarantee of uniqueness in this more general formulation. There may be multiple stable solutions, but there must be at least one. Also, note that since there is only one steady state in the main model which is a special case encompassed by Assumption 3, it must be stable. Further, the core result of the main model that behavioral policing amplifies disparities holds in this general formulation.

Proposition 6. *Under Assumption 3, in any stable interior steady state allocation:*
(i) w_A^* *is strictly increasing in* r_t*, and* $\frac{w_A^*}{\partial r_t} > \frac{\partial w_A^\dagger}{\partial r_t}$*, and*
(ii) If $w_A^* \neq 1/2$*, then* Δ^* *is strictly increasing in* ν*.*

Proof of Proposition 6. The general steady state condition for the single-officer model is as follows:

$$F(w_A^*; r_t, \nu) = w_A^{\text{br}}(r_t, \tilde{r}_p(w_A^*)) - w_A^* = 0 \tag{16}$$

where we explicitly write F to be a function of exogenous parameters of interest (here, r_t and ν).

Implicitly differentiating 16 with respect to r_t gives:

$$\frac{\partial w_A^*}{\partial r_t} = \frac{\frac{\partial w_A^{\text{br}}}{\partial r_t}}{1 - \frac{\partial w_A^{\text{br}}}{\partial \tilde{r}_p} \frac{\partial \tilde{r}_p}{\partial w_A^*}}$$

The denominator of the right hand side is $-\frac{\partial F}{\partial w_A^*}$ which at a stable solution must be positive, so the expression is positive. Further, we know that $\frac{\partial w_A^{\text{br}}}{\partial \tilde{r}_p} \frac{\partial \tilde{r}_p}{\partial w_A^*} > 0$, and so $\frac{\partial w_A^*}{\partial r_t} > \frac{\partial w_A^{\text{br}}}{\partial r_t} = \frac{\partial w_A^\dagger}{\partial r_t}$.

For part (ii), implicitly differentiating 16 with respect to ν gives:

$$\frac{\partial w_A^*}{\partial \nu} = \frac{\frac{\partial w_A^{\text{br}}}{\partial \tilde{r}_p} \frac{\partial \tilde{r}_p}{\partial \nu}}{1 - \frac{\partial w_A^{\text{br}}}{\partial \tilde{r}_p} \frac{\partial \tilde{r}_p}{\partial w_A^*}}$$

By the logic from above, the denominator of the right hand side is strictly positive. Then, this condition shows that $\frac{\partial w_A^*}{\partial \nu} > 0$ if and only if $\frac{\partial \tilde{r}_p}{\partial \nu} > 0$. However, from equation (5) in the main text, there are three cases: (1) $\frac{\partial \tilde{r}_p}{\partial \nu} > 0$ if $w_A^* > 1/2$, (2) $\frac{\partial \tilde{r}_p}{\partial \nu} < 0$ if $w_A^* < 1/2$ and (3) $\frac{\partial \tilde{r}_p}{\partial \nu} = 0$ if $w_A^* = 1/2$. Moreover, note that that if $\nu = 0$, then $w_A^* = w_A^\dagger$. Combining these observations: (1) if $w_A^* > 1/2$, then $|w_A^* - w_A^\dagger| = w_A^* - w_A^\dagger$ is increasing in ν since w_A^* is increasing away from $w_A^\dagger$ as ν increases, and (2) if $w_A^* < 1/2$, then $|w_A^* - w_A^\dagger| = w_A^\dagger - w_A^*$ is increasing in ν since w_A^* is decreasing away from $w_A^\dagger$ as ν increases. Finally, since (3) $w_A^* = w_A^\dagger$ for all ν if w_A^*, then we have shown that Δ^* is strictly increasing in ν if and only if $w_A^* \neq 1/2$. ∎

D.3 More General Beliefs (Multiple Officer Model)

There are several ways one could extend the definiton of non-conditioning bias to the multiple officer model. One potentially realistic change would be to assume that officers may do a better (or worse) job of adjusting for their own behavior than others' behavior when forming inferences about the p_J parameters. Formally, we could define the officer belief as:

$$\tilde{r}_{p,i}(w_A) = \frac{\frac{c_A}{\nu_i^s + (1-\nu_i^s) w_{A,i} + \nu_i^o + (1-\nu_i^o) w_{j,2}}}{\frac{c_B}{\nu_i^s + (1-\nu_i^s) w_{B,i} + \nu_i^o + (1-\nu_i^o) w_{B,j}}} \tag{17}$$

where the $\nu_i^s \in [0,1]$ represents how well the officer conditions for his own allocation and $\nu_i^o \in [0,1]$ represents how well he conditions on the other officer choice. A key feature of this more general belief is that as long as $\nu_i^s > 0$, it is increasing in $w_{A,i}$, meaning the officer's belief about

A's relative crime rate increases in how much he polices this group. Similarly, as long as $\nu_i^o > 0$, the officer's belief about the relative crime rate of group A increases in how much the other officer polices this group. So, while the the analysis is more complicated with this belief formation, the general feedback loop and spillover dynamics are present here as well.

E Nonlinear Returns to Policing

Returning to the original utility function, recall an additional way to motivate the diminishing returns assumption is that the marginal rate of crimes caught among group J decreases as w_J increases. Suppose the number of crimes caught is equal to $c_J = f(p_J w_J)$ where f is an increasing and concave function. Assume that the officers knows this functional form, but not the p_J parameters.

Knowing c_J and w_J, a fully Bayesian officer could then infer p_J by inverting the f function: $p_J = f^{-1}(c_J)/w_J$. The officer would then form a correct inference about the relative crime "rates" of the group, where the scare quotes highlight that the p_J parameters no long have a simple interpretation as the average crime rates of the groups:

$$\tilde{r}_p(0) = \frac{f^{-1}(c_A)/w_A}{f^{-1}(c_B)/w_B} = p_A/p_B$$

Note that if the officer now beliefs the relative crime rates are equal to c_A/c_B, he is making two mistakes: not adjusting for w_J, and also not accounting for the nonlinear effect of policing effort. In this case his belief about the relative prevalence of crime among members of each group (as a function of the allocation decision) becomes:

$$\frac{f(p_A w_A)}{f(p_B(w - w_A))}$$

Which, as long as f is increasing, is increasing in w_A. Unfortunately with this notion of naivety there is not a natural way to come up with an "intermediate" form of the bias.

One potentially instructive special case is if f is a power function: $f(p_A w_A) = (p_A w_A)^\alpha$, $\alpha \in (0,1)$. In this case the fully naive belief simplifies to:

$$\frac{(p_A w_A)^\alpha}{(p_B(w - w_A))^\alpha} = r_p^\alpha \left(\frac{w}{w - w_A}\right)^\alpha$$

If $r_p = 1$ this belief will be correct when $w_A = 1/2$ (and, so with no animus, the officer will again pick a correct allocation). Now when $r_p > 1$, $1 < r_p^\alpha < r_p$. So, if the officer were to allocate his time evenly between the groups, he would now *underestimate* the relative prevalence of crime among members of the group with the higher crime rate. In other words, "not understanding diminishing returns" could lead to the opposite effect as the bias we study.

Another way to model a naive officer is that he is able to "invert" the f function but does not account for the differential policing rate. Such an officer's belief becomes:

$$\frac{p_A w_A}{p_B(w - w_A)}$$

as in the baseline, so we can again define the intermediate form of naivety identically.

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