Online Appendix B for "Propaganda and Credulity"

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This document contains several extensions and other analyses referenced in the main version of "Propaganda and Credulity"

1 Example with No Pure Strategy Equilibrium

By the convexity assumption, c' is increasing, but d may be decreasing, so it is possible to have multiple solutions to the equilibrium condition. Figure 1 shows an example where this is true: the top left panel plots the right-hand side of the equilibrium condition, which crosses 1 twice. However, as the top left panel shows, neither of these corresponds to a pure strategy equilibrium, as the best response function does not cross the 45 degree line. The bottom two panels indicate why: when the low level of manipulation is expected, picking that level corresponds to a local maximum, but the government prefers to deviate to a higher manipulation level. Conversely, when the higher level is expected, it is a gain a local maximum, but the global maximum is at a lower level of manipulation.

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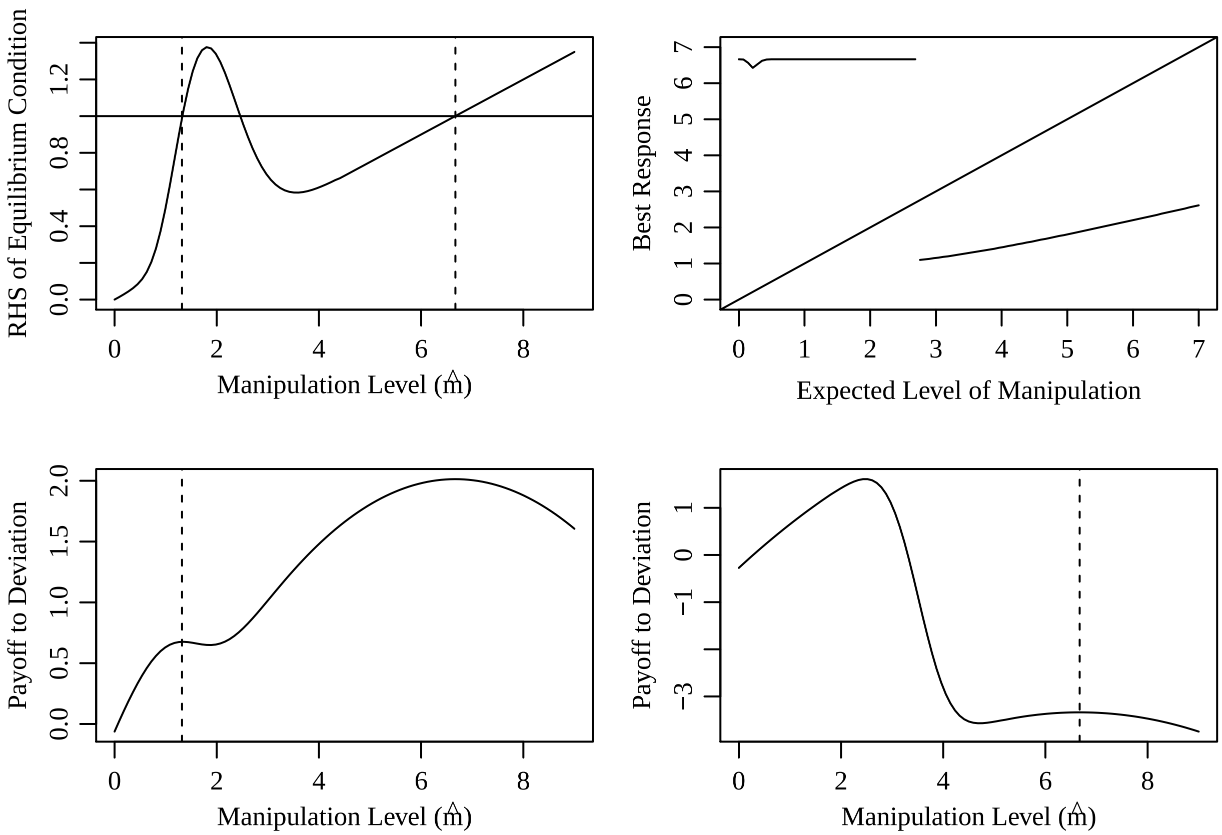
Figure 1: Example with multiple solutions to equilibrium condition, but neither correspond to a pure strategy equilibrium.



2 General Results for the Citizen Stage

The signal structure in the main text is stark: either the public signal is exactly equal to the true performance or it is upwardly distorted by a fixed amount m. To show the main results are not sensitive to these assumptions, I derive results for more general joint distributions of θ, s, and ω while maintaining that the government presence of manipulation is still binary (i.e., $\omega \in \{0, 1\}$), then derive results where the levels of manipulation vary more incrementally.

In all of this analysis I treat the government manipulation as exogenous.

2.1 More General Signal Structures

Consider any information structure where the conditional expected beliefs about the government's performance ($\mathbb{E}[\theta|s, \omega = 0]$ and $\mathbb{E}[\theta|s, \omega = 1]$) are well defined. The best response functions are still $BR(I, s) = \lambda\mathbb{E}[\overline{a}|I, s] + (1 - \lambda)\mathbb{E}[\theta|I, s]$. Solving this system of equations gives:

$$a^*_\emptyset(s) = \mathbb{E}[\theta|s, \emptyset] \tag{1}$$

$$a^*_0(s) = \frac{1 - \lambda + (1 - \overline{q}(s))\epsilon\lambda}{1 - \lambda + \epsilon\lambda}\mathbb{E}[\theta|s, \omega = 0] + \frac{\overline{q}(s)\epsilon\lambda}{1 - \lambda + \epsilon\lambda}\mathbb{E}[\theta|s, \theta = 1] \tag{2}$$

$$a^*_1(s) = \frac{1 - \lambda + \overline{q}(s)\epsilon\lambda}{1 - \lambda + \epsilon\lambda}\mathbb{E}[\theta|s, \omega = 1] + \frac{(1 - \overline{q}(s))\epsilon\lambda}{1 - \lambda + \epsilon\lambda}\mathbb{E}[\theta|s, \theta = 0] \tag{3}$$

The uninformed type always picks the action equal to their expectation of ω. The informed types both pick an action that is a weighted average of their expectation of θ given s and what their expectation would be if they knew ω took on the opposite value. Three of the core properties of the baseline model generalize as follows:

Proposition 1. *For any joint distribution of θ, s, and ω such that $\omega \in \{0, 1\}$:*
(i) As $\lambda \to 1$, the action taken by the informed types approaches that of the uninformed types,
(ii) As $\lambda \to 1$ the action taken by the informed type in the fully credulous equilibrium ($q \to 0$) approaches $\mathbb{E}[\theta|s, \omega = 0]$, and
(iii) If further $\mathbb{E}[\theta|s, \omega = 1] < \mathbb{E}[\theta|s, \omega = 0]$, the action taken by the type that knows $\omega = 1$ is strictly increasing in λ and ϵ, and strictly decreasing in q.

Proof Follows immediately from equations 1-3.

Part (i) generalizes the blind leading the sighted result: when coordination motives matter most, the behavior of all citizens approaches the expected belief of the uninformed types. When the uninformed types believe it is unlikely that $\omega = 1$, then the behavior of the informed types approaches the expected belief if $\omega = 0$ even for those who know $\omega = 1$. Part (iii) states that as long as those who know $\omega = 1$ believe the government is weaker for a fixed reported performance, they behave more favorably towards the government when the coordination motive is high, there are more credulous citizens, and the credulous citizens think it is less likely that the signal is manipulated.

2.2 More General Willingness/Ability to Manipulate

In the above there are always two types of government in terms of the ability to manipulate information. Now suppose ω is drawn from any distribution with finite expectation q (restricting $q \in [0, 1]$ makes the interpretation closer to main model, but this does not affect the analysis), where $\mathbb{E}[\theta|s, \omega]$ is always finite as well. Again, a fraction $1 - \epsilon$ of the citizens observe ω while fraction ϵ do not.

Write the action by the uninformed types $a_{\emptyset}(s)$ and the action take by a citizen observing ω $a_{\omega}(s)$. Following a similar computation to the main model, this results in the following:

Proposition 2. *For any distribution of* ω*:*

(i) As $\lambda \to 1$, $a^*_{\omega}(s) \to \mathbb{E}[\omega|s, \emptyset]$ *for all* ω*,*

(ii) For types such that $\mathbb{E}[\theta|s, \omega] < \mathbb{E}[\theta|s, \emptyset]$*, the equilibrium action is increasing in* λ *and* ϵ*.*

Proof The payoffs are the same as above, so the action taken by the informed types must meet:

$$a^*_{\omega}(s) = \lambda(\epsilon a^*_{\emptyset}(s) + (1 - \epsilon)a^*_{\omega}(s)) + (1 - \lambda)\mathbb{E}[\theta|s, \omega]$$

Rearranging gives:

$$a^*_{\omega}(s) = \frac{\lambda \epsilon a^*_{\emptyset}(s) + (1 - \lambda)\mathbb{E}[\theta|s, \omega|]}{1 - \lambda(1 - \epsilon)} \quad (4)$$

The optimal strategy for the uninformed type must meet:

$$\begin{aligned} a^*_\emptyset(s) &= \lambda(\epsilon a^*_\emptyset(s) + (1-\epsilon)\mathbb{E}[a_\omega(s)|s,\emptyset]) + (1-\lambda)\mathbb{E}[\omega|s,\emptyset] \\ &= \lambda\left(\epsilon a^*_\emptyset(s) + (1-\epsilon)\frac{\lambda\epsilon a^*_\emptyset(s) + (1-\lambda)\mathbb{E}_\omega[\mathbb{E}[\theta|s,\omega]]}{1-\lambda(1-\epsilon)}\right) + (1-\lambda)\mathbb{E}[\theta|s,\emptyset] \end{aligned}$$

and since $\mathbb{E}_\omega[\mathbb{E}[\theta|s,\omega]] = \mathbb{E}[\theta|s,\emptyset]$, rearranging gives:

$$a^*_\emptyset(s) = \mathbb{E}[\theta|s,\emptyset]$$

So, as in all of the previous analysis, the uninformed types play an action equal to their posterior belief about the government's performance.

Plugging this back into equation 4 gives:

$$a^*_\omega(s) = \frac{\lambda\epsilon\mathbb{E}[\theta|s,\emptyset] + (1-\lambda)\mathbb{E}[\omega|s,\omega]}{1-\lambda(1-\epsilon)} \tag{5}$$

So, when the coordination motive is strong, informed types still mimic the behavior of the uninformed types, even if there are many potential real levels of manipulation. Further, whichever types have a lower assessment of the regime performance based on their knowledge of ω will pick higher actions when there are more credulous citizens or a stronger coordination motive.

3 Further Extensions

Finally, I briefly describe several other extensions to the model.

3.1 Alternative Notions of Credulity

In all of the previous analysis the credulous citizens are aware of the fact some have more information than they do. A seemingly stronger notion of credulity is that some subset of citizens not only has an incorrect belief about the probability that they are being lied to, but also thinks everyone shares the same incorrect belief. When formalizing this notion credulity in a natural way, equilibrium behavior is *identical* to the main model.

In particular, suppose the uninformed citizens think that everyone else also believes the government manipulates with probability q, and the informed types know that fraction ϵ have this (incorrect) belief. If the credulous types are unaware that others have better information than them and conjecture that all use strategy $a_{\emptyset}(s)$, to be playing mutual best responses this action must satisfy:

$$a_{\emptyset}(s) = \lambda a_{\emptyset}(s) + (1 - \lambda)(s - \overline{q}(s)m)$$

Which implies $a^*_{\emptyset}(s) = s - \overline{q}(s)m$, as in the main model. Given this strategy, the optimal strategies for the informed types (who do know that there are credulous types) are also identical to those in the citizen stage (when the citizens form a correct conjecture about m). The equilibrium behavior is also identical if assuming that the uniformed types are "behavioral" types who always pick an action equal to their expected belief about θ without regards to what others do.[1]

3.2 Government (Partially) Informed about Performance

If the regime knows θ, then their strategy is a mapping from θ and ω to a manipulation level. Since manipulation has no impact on the signal and is costly if $\omega = 0$, then as in the main analysis only those with $\omega = 1$ will choose a positive manipulation level. Now write this $m(\theta)$.

To see there is a pooling equilibrium in the case of full credulity, suppose all with $\omega = 1$ choose m^* which solves $c'(m^*) = 1$. The informed types then know that $\theta = s - m^*$ with probability 1,

[1] The results are also generally robust to allowing the credulous citizens to form beliefs in a non-Bayesian manner, provided they all use the same updating rule which is common knowledge. The only result in the citizen model that relies on the belief being formed by Bayes' rule (and f being log-concave) is the total responsiveness to manipulation decreasing in m. This would not hold, if, for example, q is constant in s, in which case the responsiveness is not a function of m.

and the uninformed type belief that $\omega = 1$ is always 0. So, the payoff for choosing manipulation level m as a function of θ is:

$$u_G(m; \theta, \hat{m}) = \theta + m - \left(1 - \frac{\epsilon}{1 - \lambda(1-\epsilon)}\right)\hat{m} - c(m).$$

While this is a function of θ and $\hat{m}$, the derivative with respect to m is always 1, as so the optimal manipulation level for all types is characterized by $c'(m^*) = 1$. A similar logic holds if the government is partially informed about their performance.

Now consider the case without full credulity. In order to make the citizen inferences as easy as possible, a natural equilibrium to check for is one where $s = \theta + \omega m(\theta)$ is invertible. Suppose the citizens conjecture that the government uses manipulation strategy $\hat{m}(\theta)$ In this case, the beliefs of the informed types are:

$$\mathbb{E}[\theta | s, \omega = 0; \hat{m}] = s$$
$$\mathbb{E}[\theta | s, \omega = 1; \hat{m}] = \theta^{-1}(s)$$

where θ^{-1} is the inverse function of s (conditional on manipulation). An uninformed type observing s knows that either $\omega = 0$ and $s = \theta$ or $\omega = 1$ and $\theta + \theta^{-1}(s)$. So the posterior belief that $\omega = 1$ is:

$$\overline{q}(s) = \frac{qf(\theta^{-1}(s))}{qf(\theta^{-1}(s)) + (1-q)f(s)}$$

as so the expected belief about θ given s for the uninformed types is:

$$\mathbb{E}[\theta | s; \hat{m}] = \overline{q}(s)\theta^{-1}(s) + (1 - \overline{q}(s))s$$

Applying equations 1-3, the actions in the equilibrium with $q-$credulity are:

$$a^*_\emptyset(s) = \overline{q}(s)\theta^{-1}(s) + (1 - \overline{q}(s))s \tag{6}$$

$$a^*_1(s) = \frac{1 - \lambda + \overline{q}(s)\epsilon\lambda}{1 - \lambda + \epsilon\lambda}\theta^{-1}(s) + \frac{(1 - \overline{q}(s))\epsilon\lambda}{1 - \lambda + \epsilon\lambda}s \tag{7}$$

and the average action taken as a function of s is:

$$\begin{aligned}\overline{a}(s) &= \epsilon a^*_\emptyset(s) + (1 - \epsilon)a^*_1(s) \\ &= (1 - w_0(s))\theta^{-1}(s) + w_0(s)s\end{aligned}$$

where $w_0(s) = \frac{\epsilon(1-\overline{q}(s))}{1-\lambda+\lambda\epsilon}$

The government payoff for choosing manipulation level m when the true performance is θ is:

$$u_g(m, \theta) = \overline{a}(\theta + m) - c(m)$$

giving first order condition for type θ:

$$c'(m) = \overline{a}'(\theta + m) \tag{8}$$

The right hand side is the increase in the average action when picking a higher manipulation level:

$$\overline{a}'(\theta + m) = (1 - w_0(\theta + m))\frac{\partial\theta^{-1}(\theta + m)}{\partial m} + w_0(\theta + m) + (\theta + m - \theta^{-1}(\theta + m))\frac{\partial w_0(\theta + m)}{\partial m}$$

If there exists a m that solves equation 8 for all θ then there is a separating equilibrium of this form.

3.3 Second Order Credulity

Finally, I show that similar results for the citizen stage hold in a model where no citizens are actually credulous, but some are *second-order credulous*, meaning they incorrectly believe that others are credulous. To simplify, I do not directly model the regime choice here, taking the manipulation level conditional on it occurring exogenously as $m > 0$.

As with (first-order) credulity, I formalize this notion as the limiting case of a BNE where some citizens hold an "incorrect" belief, now about whether credulous citizens exist. In this section, the state variable that indicates whether the government manipulates is $\omega_0 \in \{0, 1\}$, i.e., the public signal is $s = \theta + \omega_0 m$. There is now a second state variable $\omega_1 \in \{0, 1\}$ which indicates whether some group of citizens are uninformed about the government's ability to manipulate. In particular, $\omega_1 = 1$ indicates that there exists a fraction $\epsilon_1 > 0$ of citizens who do not observe ω_0 and believe $\omega_0 = 1$ with probability q_0. When $\omega_1 = 0$, all of the citizens know whether the government can manipulate, i.e., all directly observe ω_0.

A (common knowledge) fraction of citizens $\epsilon_2 \in (0, 1-\epsilon_1)$ directly observe ω_0 but not ω_1. That is, some citizens know whether or not the government can manipulate but are not sure whether all of their fellow citizens also know this. Those that do not directly observe ω_1 believe that $Pr(\omega_1 = 1) = q_1$. The primitive random variables θ, ω_0, and ω_1 are independent.

There are now seven possible information sets for each signal realization. Denote the information set (not including the signal) for those who observe neither ω_0 nor ω_1 with $I = \emptyset$. There are two information sets for those who observe ω_0 but not ω_1, write these $I = \omega_0$. For example, the action taken by those who know the government can manipulate ($\omega_0 = 1$) but are unsure if credulous citizens exist (i.e., don't observe ω_1) is denoted $a_1(s)$. Finally, there are four information sets for those who observe ω_0 and ω_1, denoted $I = \omega_0, \omega_1$. For example, the action taken by the type that knows the government can manipulate and that everyone is aware of this ($\omega_0 = 1$ and $\omega_1 = 0$) is written $a_{1,0}(s)$.

The best response for each type is again $\lambda \mathbb{E}[\overline{a}|I, s] + (1 - \lambda)\mathbb{E}[\theta|I, s]$. Given s, types observing

$\emptyset$ believe that the expected performance is $s - \overline{q}_0(s)m$, where $\overline{q}_0(s)$ is the posterior belief that $\omega_0 = 1$ given s, which is equal to $\overline{q}(s)$ in the previous section. Types observing $\omega_0 = 0$ know the government's performance is s, and those observing $\omega_0 = 1$ know the government's performance is $s - m$.

As above, the expected value of $\overline{a}$ depends on the strategies used by other citizens, as well as their belief about the composition of the population. As shown in the appendix, the best response functions for the seven information sets generate a linear system of equations with a unique solution.

The equilibrium outcome with second-order credulity is the case where the government can manipulate and everyone knows this, but some citizens believe that credulous citizens exist:

Definition The *equilibrium with* (q_0, q_1)*-second-order credulity* is the equilibrium outcome when $\omega_0 = 1$ and $\omega_1 = 0$. The *equilibrium outcome with full second-order credulity* is the outcome when $\omega_0 = 1$, $\omega_1 = 0$, $q_0 = 0$ and $q_1 = 1$.

The responsiveness of the citizens in this equilibrium share similar characteristics to the equilibrium with first-order credulity:

Proposition 3. *In the equilibrium with* (q_0, q_1)*-second-order credulity:*

i) $\overline{r}_1 > 0$ *and* $\overline{r}_{1,0} > 0$

ii) $\overline{r}_1$ *and* $\overline{r}_{1,0}$ *are strictly increasing in* λ, ϵ_1, ϵ_2, *and* q_1*; strictly decreasing in* q_0*; and provided* f *is log-concave, decreasing in* m*; and*

iii) as $\lambda \to 1$, $\overline{r}_1 \to 1 - \overline{\overline{q}}_0$ *and* $\overline{r}_{1,0} \to 1 - \overline{\overline{q}}_0$.

Proof For the $\mathbb{E}[\overline{a}|I, s]$ terms, first consider those who observe neither ω_0 nor ω_1, i.e. $I = \emptyset$. This type who "knows nothing" is not completely ignorant, as he infers from his own existence that there are other equally uninformed types ($\omega_1 = 1$), but does not know ω_0. So, he knows that fraction ϵ_1 will pick action $a_\emptyset(s)$. When $\omega_0 = 0$ (which occurs with probability $1 - \overline{q}_0(s)$), fraction ϵ_2 pick action $a_0(s)$ and fraction $1 - \epsilon_1 - \epsilon_2$ pick action $a_{0,1}(s)$. When $\omega_0 = 1$ (which occurs with probability $\overline{q}_0(s)$), fraction ϵ_2 pick action $a_1(s)$ and fraction $1-\epsilon_1-\epsilon_2$ use the strategy $a_{1,1}(s)$. So, the expected action for those observing $\emptyset$ is:

$$\begin{aligned}\mathbb{E}[\overline{a}|\emptyset] =& \epsilon_1 a_\emptyset(s) + (1-\overline{q}_0(s))(\epsilon_2 a_0(s) + (1-\epsilon_1-\epsilon_2)a_{0,1}(s)) \\ &+ \overline{q}_0(s)(\epsilon_2 a_1(s) + (1-\epsilon_1-\epsilon_2)a_{1,1}(s))\end{aligned}$$

Similar calculations give the expected average action for the types that observe ω_0 but not ω_1 are:

$$\begin{aligned}\mathbb{E}[\overline{a}|\omega_0 = i] =& (1-q_1)[\epsilon_2 a_i(s) + (1-\epsilon_2)a_{i,0}(s)] \\ &+ q_1[\epsilon_1 a_\emptyset(s) + \epsilon_2 a_i(s) + (1-\epsilon_1+\epsilon_2)a_{i,1}(s)]\end{aligned}$$

for $i \in 0, 1$, and for those that observe ω_0 and ω_1:

$$\begin{aligned}\mathbb{E}[\overline{a}|\omega_0 = i, \omega_1 = 0] =& \epsilon_2 a_i(s) + (1-\epsilon_2)a_{i,0}(s) \\ \mathbb{E}[\overline{a}|\omega_0 = i, \omega_1 = 1] =& \epsilon_1 a_\emptyset(s) + \epsilon_2 a_i(s) + (1-\epsilon_1-\epsilon_2)a_{i,1}(s)\end{aligned}$$

Plugging these values into the best response function gives a system of seven linear equations with seven unknowns, with the following solution:

$$
\begin{aligned}
a^*_\emptyset(s) &= s - \overline{q}_0(s)m \\
a^*_0(s) &= s - \frac{\overline{q}_0(s)q_1\epsilon_1\lambda(1-\lambda(1-\epsilon_2))}{(1-\lambda)(1-\lambda(1-\epsilon_1-\epsilon_2))+\lambda^2 q_1\epsilon_1\epsilon_2}m \\
a^*_1(s) &= s - \frac{1-\lambda(2-\epsilon_1+q_1\epsilon_1-\overline{q}_0(s)q_1\epsilon_1-\epsilon_2)+\lambda^2(1-\epsilon_2-\epsilon_1(1-q_1+\overline{q}_0(s)q_1(1-\epsilon_2)))}{(1-\lambda)(1-\lambda(1-\epsilon_1-\epsilon_2))+\lambda^2 q_1\epsilon_1\epsilon_2}m \\
a^*_{0,0}(s) &= s - \frac{\overline{q}_0(s)q_1\epsilon_1\epsilon_2\lambda^2}{(1-\lambda)(1-\lambda(1-\epsilon_1-\epsilon_2))+\lambda^2 q_1\epsilon_1\epsilon_2}m \\
a^*_{0,1}(s) &= s - \frac{\overline{q}_0(s)\epsilon_1\lambda(1-\lambda(1-q_1\epsilon_2))}{(1-\lambda)(1-\lambda(1-\epsilon_1-\epsilon_2))+\lambda^2 q_1\epsilon_1\epsilon_2}m \\
a^*_{1,0}(s) &= s - \frac{1-\lambda(2-\epsilon_1-\epsilon_2)+\lambda^2(1-\epsilon_2-\epsilon_1(1-\overline{q}_0(s)q_1\epsilon_2))}{(1-\lambda)(1-\lambda(1-\epsilon_1-\epsilon_2))+\lambda^2 q_1\epsilon_1\epsilon_2}m \\
a^*_{1,1}(s) &= s - \frac{1-\lambda(2-\overline{q}_0(s)\epsilon_1+\epsilon_2)+\lambda^2(1-\epsilon_2-\overline{q}_0(s)\epsilon_1(1-q_1\epsilon_2))}{(1-\lambda)(1-\lambda(1-\epsilon_1-\epsilon_2))+\lambda^2 q_1\epsilon_1\epsilon_2}m
\end{aligned}
$$

In the equilibrium with second-order credulity, there are two types of citizens: those who know the government manipulates and that there are no credulous citizens, and those that know the government manipulates but think there might be (or, with full second-order credulity, certainly are) some credulous citizens. Let $\overline{r}_{1,0}$ and $\overline{r}_1$ be the average responsiveness to propaganda for these types, respectively (again, see the appendix for a full derivation).

The average responsiveness to manipulation for the $I = 1$ and $I = 1, 0$ types in the equilibrium with second order credulity is equal to the expectation (with respect to θ) of one minus the coefficient on the m terms for the type's equilibrium action:

$$\overline{r}_1 = 1 - \frac{1-\lambda(2-\epsilon_1+q_1\epsilon_1-\overline{\overline{q}}_0 q_1\epsilon_1-\epsilon_2)+\lambda^2(1-\epsilon_2-\epsilon_1(1-q_1+\overline{\overline{q}}_0 q_1(1-\epsilon_2)))}{(1-\lambda)(1-\lambda(1-\epsilon_1-\epsilon_2))+\lambda^2 q_1\epsilon_1\epsilon_2} \tag{9}$$

$$\overline{r}_{1,0} = 1 - \frac{1-\lambda(2-\epsilon_1-\epsilon_2)+\lambda^2(1-\epsilon_2-\epsilon_1(1-\overline{\overline{q}}_0 q_1\epsilon_2))}{(1-\lambda)(1-\lambda(1-\epsilon_1-\epsilon_2))+\lambda^2 q_1\epsilon_1\epsilon_2} \tag{10}$$

where $\overline{\overline{q}}_0 = \mathbb{E}_\theta[\overline{q}_0(s)]$. The comparative statics of the proposition follow from differentiating equations 9 and 10, and the fact that $\overline{\overline{q}}_0$ is increasing in q and m by propositions 1 and 4 in the main text. ∎

The intuition behind this result is that the types who (incorrectly) believe there are credulous types mimic their behavior for the reasons described above. And, since the fully informed types know the partially informed types mimic the behavior of non-existent credulous citizens, they must do so as well. As a result, **everyone can end up acting as if they believe the government even if none actually do.** This result suggests that any lack of common knowledge about propaganda in the direction of believing the government can't manipulate, believing some others think the government can't manipulate, believing some others think that some others think that the government can't manipulate, etc., may be enough to induce citizens to herd on acting as if they believe the government when the coordination motive is high.

This result highlights the possibility that propaganda can affect citizen behavior even if it affects none of their beliefs. What matters is that citizens *anticipate* that some other citizens beliefs are affected by propaganda. Anticipated changes in behavior due to propaganda can become self-fulfilling even if they are driven by an incorrect conjecture about changes in beliefs. More generally, a wide variety of "incorrect" political and economic beliefs can be widely stated if not widely believed, in particular if there is uncertainty about what others actually know and think.